

ESSAYS ON INTERNATIONAL INVESTMENT
ALLOCATION: THE ROLE OF LIQUIDITY,
ASYMMETRIC INFORMATION AND BELIEFS

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ESSAYS ON INTERNATIONAL INVESTMENT ALLOCATION: THE ROLE OF LIQUIDITY, ASYMMETRIC INFORMATION AND BELIEFS

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My dissertation addresses topics in international finance. The first chapter develops a theoretical framework to analyze the composition of foreign investment during liquidity crises. The second chapter examines the role of adverse selection and liquidity in the breakdown of trade during crisis. The third chapter studies the equity home bias puzzle in a decision-theoretic framework.

In the first chapter, I develop a two-country model that analyzes the composition of capital flows (direct vs. portfolio) across two countries in the presence of heterogeneity in liquidity risk and asymmetric information about investment productivity. Direct investment is characterized by higher profitability and private information about investment productivity, while portfolio investment provides greater risk diversification. I demonstrate the possibility of multiple equilibria due to strategic complementarities in choosing direct investment. I analyze the effect of an increase in the liquidity risk on the composition of foreign investment. If there is a unique equilibrium, then higher liquidity risk leads to a higher level of foreign direct investment (FDI). If, however, there are multiple equilibria, higher liquidity risk may lead to the opposite effect, a decline of FDI. In this case, an outflow of FDI is induced by self-fulfilling expectations. The dual effect of increased liquidity risk on capital flows can be related to empirically observed patterns of foreign investment during liquidity crises. Furthermore, my model offers a liquidity-based explanation for the phenomenon of bilateral FDI flows among developed countries,

and one-way FDI flows from developed to developing countries.

In the second chapter, I present a model that illustrates how adverse selection in financial markets can lead to increased asset price volatility and possibly to a breakdown of trade. The asymmetric information about asset returns generates the Akerlof's lemons problem, where buyers do not know whether the asset is sold because of its low quality or because the seller has experienced a sudden need for liquidity. The adverse selection can lead to equilibrium with no trade, reflecting the buyers' belief that most assets that are offered for sale are of low quality. I analyze the role of market liquidity and beliefs about likelihood of a crisis in amplifying the effect of adverse selection.

In the third chapter, I apply the smooth model of decision making under ambiguity to study the equity home bias puzzle. I show that difference in beliefs about perceived uncertainty, characterized by optimism or overconfidence, can significantly contribute to the explanation of the equity home bias observed in the data. I examine how ambiguity about the distribution of asset returns affects equilibrium prices and equity holdings in a two-country CARA-normal setting. All investors possess the same information about the set of possible states and the corresponding returns distribution in each state, but they have different beliefs about the likelihood of these states. In this setting, optimism and overconfidence refer to distorted beliefs about the expected mean and the dispersion of the asset returns distribution, respectively. I analyze and quantify the effects of optimism and overconfidence on asset prices and asset holdings when investors are ambiguity averse. Furthermore, I show that the equity home bias is larger in countries with smaller market capitalization.

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BIOGRAPHICAL SKETCH

Koralai Kirabaeva was born on April 30, 1979 in Moscow, Russia (then the Soviet Union). She attended the Moscow State University, where she graduated in 2000 with a Bachelor's Degree in Economics. In 2001-2002 Koralai pursued an M.A. in Economics at Carleton University in Canada. She then returned to Moscow to work at ABN Amro Bank (Moscow branch) as a foreign exchange market manager in the Treasury Department.

In 2003, Koralai began her graduate studies in Economics at Cornell University. Her primary research interests have been International Finance, Financial Economics and Macroeconomics. Koralai has been advised by Prof. Assaf Razin and Prof. Karl Shell. Her thesis committee also includes Prof. David Easley, Prof. Eswar Prasad and Prof. Viktor Tsyrennikov. Following her candidacy exam in March 2008, she received an M.A. in Economics from Cornell. She expects to graduate with a Ph.D. in August 2009. She has accepted a research position in the Financial Markets Department at the Bank of Canada starting from September 2009.

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CHAPTER 1

INTERNATIONAL CAPITAL FLOWS AND LIQUIDITY CRISES

1.1 Introduction

The two major types of international equity holdings are foreign direct investments (FDI) and foreign portfolio investments (FPI). Liquidity crises may be associated with an outflow of FPI and a simultaneous inflow of FDI, e.g., the 1994 crisis in Mexico and the late 1990s crisis in South Korea.¹ This behavior reflects the *fire-sale FDI* phenomenon when domestic companies and assets are acquired by foreign investors at fire-sale prices. However, there is evidence that some liquidity crises have been accompanied by an outflow foreign investment, including FDI, e.g., the 2001 crisis in Argentina. Some theoretical literature argues that a liquidity crunch may induce and aggravate a real crisis, leading to an exit of foreign investors.² The following question emerges: why during some liquidity crises is there an inflow of FDI while some others are accompanied by an outflow of FDI?

In this paper, I develop a model which suggests an explanation of why FDI flows exhibit such divergent behavior during liquidity crises. This paper presents a two-country general equilibrium model which analyzes the composition of investment (direct vs portfolio) across two countries in the presence of heterogeneity in liquidity risk and asymmetric information about the investment productivity.

The characteristic feature of direct investment is concentrated ownership and control which provides access to private information about investment productiv-

¹Krugman [59], Aguiar and Gopinath [4], Acharya, Shin, and Yorulmazer [1]

²Aghion, Bacchetta, and Banerjee [3], Chang and Velasco [18], and Caballero and Krishnamurthy [17].

ity³ and results in a more efficient management.⁴ Portfolio investment represents holdings of assets which do not entail active management or control but allow for risk diversification and greater liquidity. Taking advantage of the inside information, direct investors may sell low-productive investments and keep the high-productive ones under their ownership. This generates a "lemons"⁵ problem: the buyers do not know whether the investment is sold because of its low productivity or due to an exogenous liquidity shock. Therefore, due to this information asymmetry, there is a discount on the prematurely sold direct investment (relative to the prematurely sold portfolio investment). This assumption is consistent with the evidence that there is a negative premium associated with seller-initiated block trades.⁶ The main implication of this information-based trade-off is that the choice between direct and portfolio investment is linked to the likelihood with which investors expect to get a liquidity shock (Goldstein and Razin [39]).

In my model, the agents have the Diamond-Dybvig [27] type preferences. Agents consume in period 1 or 2, depending on whether they receive a liquidity shock in period 1. The probability of an investor receiving a liquidity shock is country-specific. This probability captures the investor's exposure to the liquidity shock; I will refer to it as the liquidity risk. In period zero, investors choose how much to invest into risky long-term projects in each of the two countries, as well as the ownership type for each project (direct or portfolio). In period one, idiosyncratic liquidity shocks are realized and, subsequently, risky investments are traded in the financial market. The late consumers are the buyers in the financial

³Klein, Peek, and Rosengren [56], Kinoshita and Mody [55], Bolton and von Thadden [14], Kahn and Winton [49]

⁴Due to the agency problem between managers and owners, portfolio investments are less efficient (Goldstein and Razin [39]).

⁵Akerlof (1970)

⁶Holthausen, Leftwich, and Mayers [46], Easley, Kiefer and O'Hara [28], Easley and O'Hara [29], Keim and Madhavan [52]

market. All investment projects pay off in the second period.

This "cash-in-the-market" framework⁷ allows one to capture the effect of market liquidity (demand for risky investments in the interim period) on the investment choice. The equilibrium prices of direct and portfolio investments depend not only on their expected payoffs but also on investors' liquidity preferences and uncertainty about the investment productivity. If market is more liquid then expected gains from trading on private information are larger, since it is easier for informed traders to hide behind the liquidity traders.⁸ Therefore, in a more liquid market direct investors have higher profits from selling on private information. On the other hand, a larger fraction of direct investors leads to a less liquid market.⁹

I demonstrate that there are two types of equilibria. In the first type, only investors from the country with a lower liquidity risk choose to hold direct investment. In the second type, investors from both countries hold direct investments. In this case, there are strategic complementarities in choosing direct investment. This generates a possibility of multiple equilibria through the self-fulfilling expectations. If countries have the same fundamentals, the country with a higher liquidity risk attracts less inward foreign investment, but a larger share of it is in the form of FDI. Also, the country with a higher level of asymmetric information about investment productivity attracts more FDI relative to FPI since the marginal benefits from private information are larger.

I consider the effect of an increase in the liquidity risk on the composition of foreign investment. Such an increase results in the drying up of market liquidity as more investors have to sell their risky asset holdings. At the same time, it

⁷Similarly to Allen and Gale [7] and Bhattacharya and Nicodano [12]

⁸Easley and O'Hara [29], Kyle [60]

⁹Bolton and von Thadden [14], Maug [66]

becomes more likely that if a direct investment is sold before maturity, it is sold due to exogenous liquidity needs rather than an adverse signal about investment productivity. This reduces the adverse selection problem and therefore results in a smaller information discount on prematurely sold direct investments. This effect captures the phenomenon of fire-sale FDI during liquidity crises. If economy is in the unique equilibrium then higher liquidity risk leads to a higher level of FDI. However, if there are multiple equilibria then FDI may decline as the liquidity risk becomes higher. In this case, an outflow of FDI is induced by self-fulfilling expectations.

There are two possible interpretations of the liquidity risk in my model. One is the probability of a liquidity crisis that is unrelated to fundamentals of the economy. In fact, recent financial crises exhibit a large liquidity run component while the underlying macro fundamentals are not necessarily weak.¹⁰ Another interpretation is a measure of financial market development. In more developed financial (credit) markets it is easier for agents to borrow in case of liquidity needs, and therefore the probability of investment liquidation is smaller, whereas in developing and emerging countries access to the world capital markets is limited.¹¹ So a country with a low liquidity risk can be viewed as a developed economy, and a country with a high liquidity risk can be viewed as a developing or emerging economy. In addition to a lower liquidity risk, a developed country can be characterized by a higher expected profitability (adjusted for risk) and less asymmetric information about the productivity.

In the model, the ambiguous effect of an increase in the liquidity risk on the capital flows corresponds to the empirically observed pattern of FDI during liq-

¹⁰Chang and Velasco [18] and Acharya, Shin, and Yorulmazer [2]

¹¹Freedman and Click [35]

uidity crises. The positive effect of a higher liquidity risk on the inward FDI is consistent with the evidence documented by Krugman [59], Aguiar and Gopinath [4], and Acharya, Shin, and Yorulmazer [1]. Krugman [59] notes that the Asian financial crisis has been accompanied by a wave of inward direct investment. Furthermore, Aguiar and Gopinath [4] analyze data on mergers and acquisitions in East Asia between 1996 and 1998 and find that the liquidity crisis is associated with an inflow of FDI. Moreover, Acharya, Shin, and Yorulmazer [1] observe that FDI inflows during financial crises are associated with acquisitions of controlling stakes. At the same time, my model provides a possibility of a decrease in FDI through self-fulfilling expectations. This possibility is in line with the empirical evidence¹² as well as theoretical literature that associates liquidity crises with an exit of investors from the crisis economy even if there are no shocks to fundamentals (Aghion, Bacchetta, and Banerjee [3], Chang and Velasco [18], and Caballero and Krishnamurthy [17]).

My results are consistent with the empirical findings that countries that are less financially developed and have weaker financial institutions tend to attract more capital in the form of FDI¹³. Moreover, my model can explain the phenomenon of bilateral FDI flows among developed countries, and one-way FDI flows from developed to emerging countries.¹⁴

The paper is organized as follows. Section 2 describes the related literature. Sections 3 and 4 present the theoretical model and its analysis. Sections 5 and 6 characterize the equilibrium. Sections 7 and 8 discuss the effect of change in liquidity risk on the foreign investments. Section 9 concludes the paper. All proofs are delegated to the Appendix.

¹²Lipsey [63].

¹³Albuquerque [6], Hausman and Fernandez-Arias [43]

¹⁴Razin [70]

1.2 Related Literature

My paper is related to several papers in the literature. My model builds on the adverse selection property of FDI developed by Goldstein and Razin [39]. My model differs from their model in several aspects. I examine the portfolio choice between two types of risky investment (direct vs portfolio) and safe asset in the two-country "cash-in-the-market" framework where investors have the Diamond-Dybvig [27] type of preferences (Allen and Gale [7] and Bhattacharya and Nicodano [12]). Goldstein and Razin [39] study the choice between FDI and FPI by risk-neutral investors in the partial equilibrium setting with a concave production function. They show that investors with higher liquidity needs are more likely to choose FPI over FDI. Also, they examine the implications of production costs, transparency in the host country, and heterogeneity of foreign investors in the source country. My model examines not only the composition of foreign investment but also the level thereof. My paper complements the results in Goldstein and Razin [39] by analyzing the bilateral investments flows between two countries and, furthermore, the effect of the change in liquidity preferences in the host country on inward foreign investment.

In terms of addressing the fire-sale FDI phenomenon, this paper is related to Krugman [59], Aguiar and Gopinath [4], and Acharya, Shin, and Yorulmazer [1]. Krugman [59] points out the fire-sale FDI phenomenon and offers two possible modeling approaches. One is based on moral hazard and asset deflation. The liabilities of financial intermediaries are perceived as having an implicit government guarantee, and therefore subject to moral hazard problems. The excessive risky lending inflates the asset prices, which makes the financial intermediaries seem sounder than they actually are. During a crisis, falling asset prices make the insolvency

of intermediaries visible, leading to further asset deflation. The other explanation is based on disintermediation and liquidation, attributing the crisis to a run on financial intermediaries. Such a run can be set off by self-fulfilling expectations. Aguiar and Gopinath [4] propose a model where foreign investors have financial resources to acquire domestic assets and superior technology. Acharya, Shin, and Yorulmazer [1] address the fire-sale FDI phenomenon from the firm's prospective. They provide an agency-theoretic framework in which during the crisis, the loss of control by domestic managers together with the lack of domestic capital result in a transfer of ownership to foreign firms.

This paper offers an alternative explanation of the fire-sale FDI phenomenon based on the adverse selection. In contrast to the explanations above, in my model a liquidity crisis may lead to a decline in FDI (through self-fulfilling expectations).

The following papers link financial crises and liquidity through models of self-fulfilling creditors' run. Chang and Velasco [18] place international illiquidity at the center of financial crises. They argue that a small shock may result in financial distress, leading to costly asset liquidation, liquidity crunch, and large drop in asset prices. Caballero and Krishnamurthy [17] argue that during a crisis self-fulfilling fears of insufficient collateral may trigger a capital outflow.

1.3 Model

I consider a model with 2 countries: A and B . There is a continuum of agents with an aggregate Lebesgue measure of unity. Let α be the proportion of investors living in country A , and the rest of the investors live in country B . There are 3 time periods: $t = 0, 1, 2$. There is only one good in the economy, and in period zero,

Table 1.1: Payoff structure.

time	0	1	2
safe asset	$1 \rightarrow$	$1 \rightarrow$	1
investment	$1 \rightarrow$	$0 \rightarrow$	\mathfrak{R}_k^i

all agents are endowed with one unit of good that can be consumed and invested.

1.3.1 Investment Technology

Agents have access to two types of constant returns technology. One is a storage technology (safe asset), which has zero net return: one unit of safe asset pays out one unit of safe asset in the next period. The safe asset is the same in both countries, and I will refer to it as "cash." The other type of technology is a long-term risky investment project (also called risky asset). In period two, the risky investment in project i has a random idiosyncratic payoff \mathfrak{R}^i per unit of investment which represents idiosyncratic investment productivity. Each investor i has a choice of starting his own investment project i by investing a fraction of his endowment, so each project has only one owner; the productivity realizations are independent across investments and across countries. Table 1.1 summarizes the payoff structure.

The investment productivity of each project \mathfrak{R}_k^i in country $k \in \{A, B\}$ is a independent realization of normal distribution $N(R_k, \sigma_k^2)$ with mean R_k and variance σ_k^2 .¹⁵ The productivity mean R_k is a random variable that takes two values: a low value R_{kl} with probability π_k and a high value R_{kh} with probability $(1 - \pi_k)$.¹⁶ (For each investment project in country k , nature picks the mean

¹⁵More precisely, all portfolio investments have the same productivity mean \bar{R}_{pk} , and all direct investments have the same productivity mean $\bar{R}_{dk} > \bar{R}_{pk}$, as discussed in section 1.3.3

¹⁶In addition, the probability π_k of investment project to be less productive depends on the type of ownership: the direct investment is less likely to have low mean productivity than the portfolio investment, i.e., $\pi_{dk} < \pi_{pk}$ (discussed in Section 3.3).

R_k where $R_k \in \{R_{kl}, R_{kh}\}$.¹⁷ The expected productivity mean is denoted by $\bar{R}_k = \pi_k R_{kl} + (1 - \pi_k) R_{kh}$ with $\bar{R}_k > 1$. All parameters of the productivity distribution are country-specific, with \bar{R}_k representing the expected profitability of investment project and σ_k^2 capturing the investment risk in country k .

Agents can invest their endowment in investment projects at home (domestic investment) and abroad (foreign investment). The holdings of the two-period risky investment can be traded in financial market at date $t = 1$.

1.3.2 Preferences

Agents consume in period 1 or 2, depending on whether they receive a liquidity shock in period 1. The probability of receiving a liquidity shock in period one is country-specific: investors in each country $k \in \{A, B\}$ have the same probability λ_k . This probability (λ_k) captures the liquidity risk in a given country. Investors who receive a liquidity shock have to liquidate their risky long-term asset holdings and consume all their wealth in period one. So they are effectively early consumers who value consumption only at date $t = 1$. The rest are the late consumers who value the consumption only at date $t = 2$. Since there is no aggregate uncertainty, λ_k is also a fraction of investors hit by a liquidity shock in country k .

Investors from country k have Diamond-Dybvig type of preferences:

$$U_k(c_1, c_2) = \lambda_k u(c_1) + (1 - \lambda_k) u(c_2) \quad (1.1)$$

where c_t is the consumption at dates $t = 1, 2$. In each period, investors have

¹⁷Informed investors are able to observe the true distribution, uninformed investors use the unconditional distribution which is the mixture of two normal distributions.

mean-variance utility

$$E[u(c_t)] = E[c_t] - \frac{\gamma}{2} \text{Var}[c_t] \quad (1.2)$$

with γ representing the degree of risk aversion¹⁸. Investors choose their asset holdings to maximize their expected utility.

Without loss of generality, I assume that country A has a smaller liquidity risk than country B , i.e., $\lambda_A < \lambda_B$.

1.3.3 Direct and Portfolio Investments

In period $t = 0$, agents decide how much of their endowment to invest in long-term risky investment projects. In a given country k , an agent can either invest directly in a single project, or become a portfolio investor investing in up to N_k projects.¹⁹

Direct investors are able manage projects more efficiently, therefore, the productivity of direct investment is more likely to be drawn from a high mean distribution than the productivity of portfolio investment, i.e., $\pi_{dk} < \pi_{pk}$. Therefore, the expected profitability of a direct investment (\bar{R}_{dk}) is higher than the expected profitability of a portfolio investment (\bar{R}_{pk}) per unit of investment.

Furthermore, in period one, direct investors in country k observe a signal about their investment productivity: the true value of productivity mean R_{dk} . Henceforth, I will refer to it as the productivity signal. Portfolio investors do not observe such productivity signal. Therefore, portfolio investors use the updating on the

¹⁸Maccheroni, Marinacci, and Rustichini [65] show that the mean-variance preferences is the special case of variational preferences, which is a representation of preferences for decision making under uncertainty. The mean-variance preferences have been used in the finance literature, for example, Van Nieuwerburgh and Veldkamp (2008).

¹⁹Due to the mean-variance preferences and idiosyncratic productivity, a portfolio investor will always choose to invest into the maximum number of projects allowed.

productivity mean in country k : $\bar{R}_{pk} = \pi_{pk}R_{kl} + (1 - \pi_{pk})R_{kh}$. The decision to become direct or portfolio investor is country-specific, i.e., it is possible to be a direct investor in one country, and a portfolio investor in another.

The advantage of direct investment is private information about the idiosyncratic investment productivity. However, it is public knowledge which investors are informed. This generates the adverse selection problem: it is not known whether direct investors sell due to a liquidity shock or because they have observed the negative productivity signal (high variance). Therefore there is an information discount on the price of direct investment at $t = 1$.

In this setting, the efficiency of direct over portfolio investment is reflected by higher expected productivity of the former: $\bar{R}_{dk} > \bar{R}_{pk}$. Also, the diversification benefits from portfolio investment are captured by allowing to invest in multiple projects in one country, which is effectively equivalent to reducing the investment variance by the factor of N_k . I abstract from the other gains of management control such as possibility of restructuring²⁰ that may lead to an increase of investment payoff from $t = 1$ to $t = 2$.

In period one, the liquidity shocks are realized, direct investors observe a signal about the productivity of their investments, and trading in financial market occurs. Investors who receive a liquidity shock supply their asset holdings inelastically. In addition, direct investors who have not received a liquidity shock but observe a negative productivity signal can sell their investments. The buyers are investors who have not received a liquidity shock in period one. Figure 1.1 represents the time line of the model.

²⁰The trade-off between efficiency gains related to corporate control and liquidity have been addressed by Bolton and von Thadden [14], Maug [66], and Holmstrom and Tirole [45].

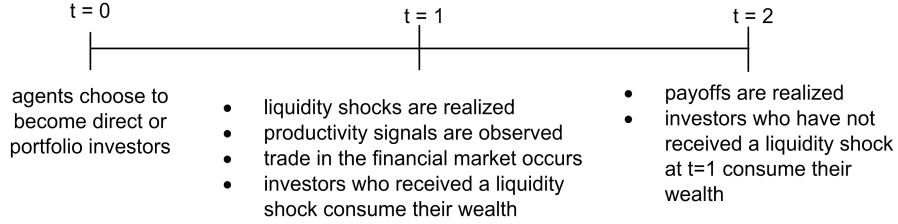


Figure 1.1: Time line.

I show that the decision between direct and portfolio investment depends on the probability of getting a liquidity shock and uncertainty about the investment productivity. Agents are more likely to choose direct investment if they are less likely to receive a liquidity shock.

1.4 Investors' Decision Problem

Agents face the following two-stage decision problem. At date $t = 0$, an agent decides whether to become a direct or a portfolio investor in each country and, correspondingly, how much of their endowment to invest in the risky long-term projects. At date $t = 1$, investors who have not received a liquidity shock, decide how much of the long-term assets they want to buy. The decision problem of portfolio and direct investors are illustrated in 1.2 and 1.3.

In period one, investors are restricted to buying either direct or portfolio investment in each country. This assumption is imposed to prevent further risk diversification. Therefore, in the equilibrium a buyer should be indifferent between buying direct or portfolio investment in a given country. Note that at period $t = 1$ there is no advantage of private information.

Let $\delta_{ik} \in [0, 1]$ be the fraction of direct investors from country i investing in

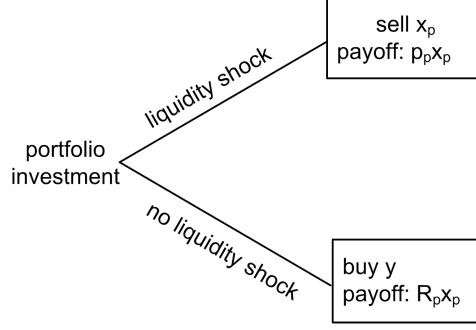


Figure 1.2: Portfolio investor's decision.

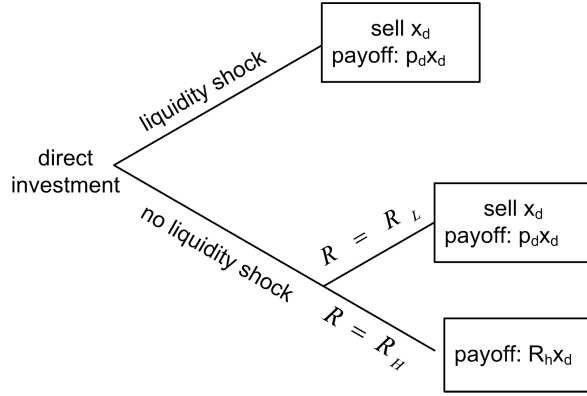


Figure 1.3: Direct investor's decision.

country k where $i, k \in \{A, B\}$. Then the fraction of direct investors investing in country k is $\delta_k = \alpha \delta_{Ak} + (1 - \alpha) \delta_{Bk}$.

The investor who buys a risky asset from a direct investor in period $t = 1$, does not know whether it is sold due to the liquidity shock or because of the low productivity mean. Buyers believe that direct investors in country k will receive a liquidity shock with probability

$$\lambda_{dk} = \frac{\alpha \delta_{Ak} \lambda_A + (1 - \alpha) \delta_{Bk} \lambda_B}{\alpha \delta_{Ak} + (1 - \alpha) \delta_{Bk}}. \quad (1.3)$$

Therefore, the buyers believe that with probability $\frac{\lambda_{dk}}{\lambda_{dk} + (1 - \lambda_{dk}) \pi_{dk}}$ direct investment

in country k is sold due to a liquidity shock, and with probability $\frac{(1-\lambda_{dk})\pi_{dk}}{\lambda_{dk}+(1-\lambda_{dk})\pi_{dk}}$ it sold because its low productivity. Hence, buyers believe that the productivity mean of the asset sold by a direct investor is low R_{kl} with probability $\frac{(1-\lambda_{dk})\pi_{dk}}{\lambda_{dk}+(1-\lambda_{dk})\pi_{dk}}$ and high R_{kh} with probability $\frac{\lambda_{dk}}{\lambda_{dk}+(1-\lambda_{dk})\pi_{dk}}$. Using Bayesian updating, the mean of the prematurely sold direct investment in country k is

$$\tilde{R}_{dk} \equiv \frac{(1-\lambda_{dk})\pi_{dk}}{\lambda_{dk}+(1-\lambda_{dk})\pi_{dk}} R_{kl} + \frac{\lambda_{dk}}{\lambda_{dk}+(1-\lambda_{dk})\pi_{dk}} R_{kh}, \quad (1.4)$$

and its variance is σ_k^2 .

Portfolio investors do not observe a productivity signal, hence they only sell their investment if they are hit by a liquidity shock. Therefore, the productivity of the prematurely sold portfolio investment in country k has mean \bar{R}_{pk} and variance σ_k^2/N_k . Since investment productivity is idiosyncratic, there is no updating on the productivity variance of portfolio investment based on the direct investors selling.

Several assumptions are imposed on the parameters $(R_{kl}, R_{kh}, \sigma_k^2, \pi_{dk}, \pi_{pk}, N_k)$ of the productivity distribution for each country k ²¹:

Assumption 1. In the absence of private information, investors are indifferent between holding direct and portfolio investment. This assumption implies that benefits from diversification are perfectly offset by benefits from management efficiency resulting in the higher expected productivity.

Assumption 2. At $t = 0$, all investors invest some but not all of their endowment in risky projects.

Assumption 3. At $t = 1$, investors' aggregate demand for risky assets is less than his safe asset holdings.

²¹See section A.1 of the Appendix

The investors from country $i \in \{A, B\}$ choose their optimal investment holdings in each country $k \in \{A, B\}$ at date $t = 0$ to maximize their expected utility. Denote by x_{dk}^i the demand for direct investment at $t = 0$ by an investor from country i . Similarly, denote by x_{pk}^i the total demand for portfolio investment at $t = 0$ by an investor from country i (this demand is divided equally among N_k projects).

At date $t = 1$, uncertainty about the liquidity shock is resolved and all investors observe the total proportion of early consumers, however, their identity is private information. Denote the prices of direct and portfolio investments in country $k \in \{A, B\}$ by p_{pk} and p_{dk} , respectively. Let y_{pk} and y_{dk} be the demand for direct and portfolio investment in country k in period one. Since the liquidity shock is realized at date $t = 1$, the demands y_{pk} and y_{dk} are the same for investors from both countries (so superscript i can be omitted).²²

The demand for direct and portfolio investments in period one are given by

$$\begin{aligned} y_{pk} &= \frac{\bar{R}_{pk} - p_{pk}}{\gamma \sigma_k^2 / N_k} \\ y_{dk} &= \frac{\tilde{R}_{dk} - p_{dk}}{\gamma \sigma_k^2} \end{aligned} \tag{1.5}$$

where $k \in \{A, B\}$.²³ Since investors are restricted to buying either only direct or only portfolio investment at $t = 1$ in a given country k , the optimal demand for the risky asset is given by $y_k = \max \{y_{dk}, y_{pk}\}$.

The optimal demand for the portfolio investment in country k by an investor from country i in period $t = 0$ is given by

²²The demand for risky asset at $t = 1$ is independent from investment demand at $t = 0$ due to the mean-variance preferences and assumption 2. Since after the realization of liquidity shock, the survived investors from both countries are identical, and their demands for each type of the risky asset is the same: $y_{pk}^A = y_{pk}^B$ and $y_{dk}^A = y_{dk}^B$.

²³See Appendix A.2 for maximization problem.

$$x_{pk}^i = \frac{(\bar{R}_{pk} - 1) - \lambda_i (\bar{R}_{pk} - p_{pk})}{(1 - \lambda_i) \gamma \sigma_k^2 / N_k} \quad (1.6)$$

The optimal demand for the direct investment in country k by an investor from country i in period $t = 0$ is given by

$$x_{dk}^i = \frac{(R_{kh} - 1) - \lambda_i (R_{kh} - p_{dk})}{(1 - \lambda_i) \gamma \sigma_k^2} \quad (1.7)$$

Note that the demand for risky investment (both direct and portfolio) at $t = 0$ is a decreasing function of liquidity risk (λ_i), i.e., investors from a country with a lower liquidity risk will allocate a larger fraction of their endowment to risky assets in period zero. Also, the demand for risky investment is an increasing function of the price of the investment at $t = 1$, i.e., agents will invest a larger amount of their endowment into risky projects if the re-sale price in the next period is higher.

1.5 Equilibrium

Recall that $\delta_{ik} \in [0, 1]$ denotes the fraction of direct investors from country i investing in country k , where $i, k \in \{A, B\}$.

Given the fractions $(\delta_{ik} : i, k \in \{A, B\})$ of direct investors in the economy, prices (p_{pk}, p_{dk}) and demand functions $(x_{pk}^i, x_{dk}^i, y_k)$ for all $i, k \in \{A, B\}$, constitute a Rational Expectations Equilibrium (REE) if (i) (x_{dk}^i, y_k) (respectively, (x_{pk}^i, y_k)) maximizes the expected utility of a direct (respectively, portfolio) investor i , given the prices (p_{dk}, p_{pk}) and (ii) the market for investments clears at $t = 1$.

The overall equilibrium in the economy is given by $(\delta_{ik}, (p_{dk}, p_{pk}), (x_{dk}^i, x_{pk}^i, y_k))$

for $i, k \in \{A, B\}$.

1.5.1 Properties of Equilibrium

Property 1. *In an equilibrium, the prices satisfy $p_{dk} \leq 1$ and $p_{pk} \leq 1$.*

If the price of direct investment in country k is greater than one then agents will invest all of their endowment in this country. Then there is no safe asset holding in period one, therefore $p_{dk} > 1$ cannot be an equilibrium price. Similarly, for portfolio investment.

Property 2. *In an equilibrium, the optimal demands for portfolio and direct investments are equal:*

$$\frac{\tilde{R}_{dk} - p_{dk}}{\gamma\sigma_k^2} = \frac{\bar{R}_{pk} - p_{pk}}{\gamma\sigma_k^2/N} \quad (1.8)$$

Given the assumption that investors can buy only one type of asset in each country, the expected utilities of buying direct and portfolio investments should be equal in the equilibrium. Otherwise, all investors will only buy the investment with higher expected utility.

Property 3. *In an equilibrium, a direct investor sells his investment if he observes a negative productivity signal.*

Suppose a direct investor does not sell his investment after observing a negative signal. Then by Assumption 3, ex-ante the investor is better off by choosing the portfolio investment at $t = 0$ since he can sell it for a higher price at $t = 1$ in case of a liquidity shock.

The equilibrium prices of direct investment (p_{dk}) and portfolio investment (p_{pk})

are determined by equation (1.8) and the market clearing condition (1.9).

$$= \begin{pmatrix} (\alpha(1 - \lambda_A) + (1 - \alpha)(1 - \lambda_B)) y_k \\ \alpha \delta_{Ak} (\lambda_A + (1 - \lambda_A) \pi_k) x_{dk}^A \\ + (1 - \alpha) \delta_{Bk} (\lambda_B + (1 - \lambda_B) \pi_k) x_{dk}^B \\ + \alpha (1 - \delta_{Ak}) \lambda_A x_{pk}^A \\ + (1 - \alpha) (1 - \delta_{Bk}) \lambda_B x_{pk}^B \end{pmatrix} \quad (1.9)$$

In each country k , risky investment is supplied by the agents who received a liquidity shock or the adverse signal about investment productivity. The buyers are the agents who have not received a liquidity shock.

1.5.2 Choice between Direct and Portfolio Investments

In period $t = 0$, an investor from country i chooses to become a direct investor in country k only if his expected utility from holding direct investment is greater than or equal to his expected utility from holding portfolio investment: $EU(x_{dk}^i) \geq EU(x_{pk}^i)$. If the two utilities are equal then an investor is indifferent between holding direct or portfolio investment.

Recall that the liquidity risk in country A is less than in country B : $\lambda_A < \lambda_B$.

Lemma 1. *For any country $k \in \{A, B\}$, if some investors from country B hold direct investment in country k , i.e., $\delta_{Bk} > 0$, then all investors from country A hold direct investment in country k , i.e., $\delta_{Ak} = 1$.*

Lemma 1 follows from the fact that the demand for risky investment is a decreasing function in liquidity risk. This lemma implies that if some investors from

country A (but not all) choose to hold direct investment in country k , then none of the investors from country B hold direct investment in that country. In particular, if for investors from country A the expected utility from holding direct investment is less than the expected utility from holding portfolio investment, then only portfolio investments will be held in equilibrium.

Proposition 1 *There exist an equilibrium. For each country $k \in \{A, B\}$, there are two possible types of equilibria. In type I, $\delta_{Ak} \in [0, 1)$ and $\delta_{Bk} = 0$, i.e., only investors from country A (but not all) hold direct investment; the equilibrium of this type is unique. In type II, $\delta_{Ak} = 1$ and $\delta_{Bk} \in [0, 1]$, i.e., all investors from country A hold direct investment; there are at most three such equilibria.*

Type I equilibrium includes the (corner) equilibrium with portfolio investments only and a pooling equilibrium for investors from country A . The equilibrium of type I is unique because there is a strategic substitutability in becoming a direct investor. Therefore, there is a unique equilibrium δ_{Ak} such that if the proportion of direct investors is below δ_{Ak} then $EU(x_{dk}^A) > EU(x_{pk}^A)$, and if the proportion of direct investors is above δ_{Ak} then $EU(x_{dk}^A) < EU(x_{pk}^A)$.

Type II equilibrium includes the (corner) equilibrium with direct investments only, a pooling equilibrium for investors from country B , and the separating equilibrium where direct investments are held by investors from country A and portfolio investments are held by investors in country B .

The multiplicity of type II equilibria is based on the effect of expectations on the price of prematurely sold direct investment. On one hand, similarly to the type I equilibrium, as the fraction of direct investors δ_{Bk} increases, the price of direct investment goes down in country k , decreasing the benefits from holding

direct investment. On the other hand, the information discount on the price of direct investment depends on the probability of direct investors selling due to the negative productivity signal. If there are more direct investors with a high liquidity risk then the market believes that the probability of a direct investor selling due to a liquidity shock is higher and, therefore, the price discount on the prematurely sold direct investment is smaller. So, more investors from country B choose to hold direct investment if they believe that other investors from country B are holding direct investment. This strategic complementarity among direct investors generates multiple equilibria. If there are two or three equilibria then one of the equilibria is a separating equilibrium where all investors with a low liquidity risk hold direct investment, and all investors with a high liquidity risk hold portfolio investment.

Overall, there are five possible cases of composition of direct and portfolio investment that can occur in the equilibrium in a given country:

1. investors from both countries hold portfolio investments;
2. some investors from country A hold direct investments and others hold portfolio investments;
3. all investors from country A hold direct investments and all investors from country B hold portfolio investments;
4. some investors from country B hold portfolio investments and others hold direct investments;
5. investors from both countries hold direct investments.

Figure 1.4 illustrates the possible equilibria regions for different values of λ_A and λ_B such that $\lambda_A < \lambda_B$. Each point in the (λ_A, λ_B) plane corresponds to

a particular case of equilibria in the enumeration above, except for the points with multiple equilibria (when cases 3 and 4 occur simultaneously). Thus, each type corresponds to a region in the plane; these regions are colored distinctly and numbered accordingly. We consider three examples with the same values of $\bar{R}_h = 1.2, \bar{R}_l = 0.9, \sigma^2 = 0.1, \pi_p = \pi_d = 0.5, N = 1$ and different values of α (the fraction of investors in country A). Note that as α becomes larger the area with multiple equilibria disappears.

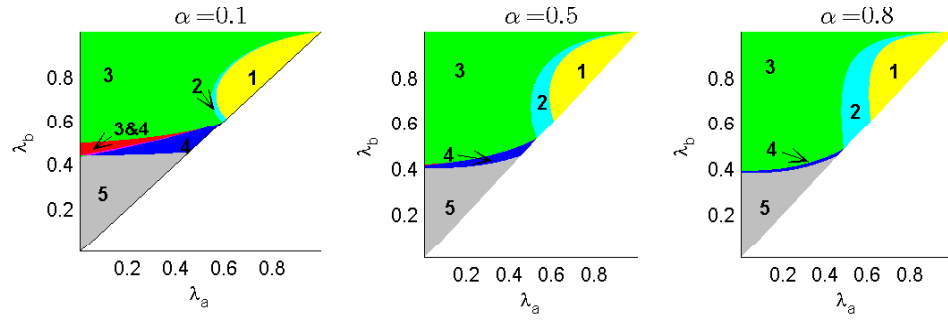


Figure 1.4: Possible equilibria regions for different values of λ_A and λ_B .

1.6 Composition of Foreign Investment

Define the foreign direct investment from country A to country B as the holdings of direct investment in country B by investors from country A : $FDI_{AB} = \alpha \delta_A x_{dB}^A$. Similarly, define the foreign portfolio investment from country A to country B as the holdings of portfolio investment in country B by investors from country A : $FPI_{AB} = \alpha (1 - \delta_A) x_{pB}^A$. Then the foreign investment from country A to country B is $FI_{AB} = \alpha \delta_A x_{dB}^A + \alpha (1 - \delta_A) x_{pB}^A$. Define FDI_{BA} , FPI_{BA} , and FI_{BA} similarly.

There are two dimensions in which the two countries may differ. One is the liquidity risk (λ_k), another is the distribution parameters of investment productivity

that represent the country's fundamentals: $(R_{kl}, R_{kh}, \sigma_k^2, \pi_{dk}, \pi_{pk}, N_k)$.

There are two possible interpretations of liquidity risk in my model. One is the probability of a liquidity crisis that is unrelated to fundamentals of the economy. Another is a measure of financial market development: in more developed financial markets it is easier for agents to borrow in case of liquidity needs, therefore the probability of investment liquidation is smaller. Accordingly, a country with a low liquidity risk can be viewed as a developed country, and a country with a high liquidity risk can be viewed as a developing or emerging economy.

Suppose the countries differ only in terms of liquidity risk and are identical with respect to productivity parameters. In this case, the country with a higher liquidity risk attracts less foreign investment, but a higher share of it in the form of FDI. Figures 1.5 illustrates the possible compositions of bilateral investment holdings in the different types of equilibria.

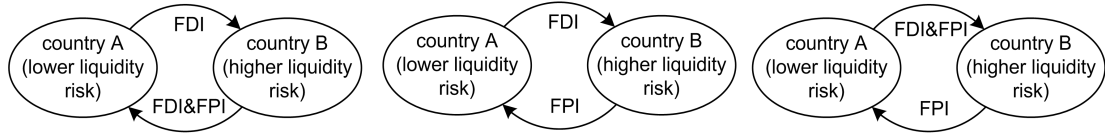


Figure 1.5: Bilateral investment holdings in different types of equilibria: type I pooling equilibrium, separating equilibrium, type II pooling equilibrium.

In addition to a lower liquidity risk, a developed country can be characterized by a higher expected payoff (adjusted for risk) and smaller benefits from private information of FDI.

Property 4. *In an equilibrium, the share of FDI from country i to country k is higher if either of the following holds: (i) efficiency gains of direct investment $(\bar{R}_{dk} - \bar{R}_{pk})$ are larger, (ii) uncertainty about investment productivity $(R_{kh} - R_{kl})$*

is larger, (iii) risk diversification opportunities (N_k) are smaller.

Both FDI and FPI holdings are larger if in the host country the expected profitability is higher and the investment risk is lower. The larger uncertainty about investment productivity positively affects the share of direct investments relative to portfolio investments since the benefits from private information are larger. If direct investment is more efficient relative to portfolio investment, then the share of direct investments is higher, which corresponds to higher equilibrium levels of δ_{Ak} and δ_{Bk} . On the other hand, larger diversification benefits from portfolio investment result in a smaller share of FDI.

My results are consistent with the empirical findings that countries that are less financially developed and have weaker financial institutions tend to attract more capital in the form of FDI. This offers a liquidity-based explanation of the phenomenon of bilateral FDI flows among developed countries and one-way FDI flows from developed to emerging countries.

Moreover, Freedman and Click [35] show that banks in developing countries maintain a high level of liquid assets, while allocating only a modest amount of funds to productive businesses through loans. They argue that this difference among developed and developing countries is due to inefficiencies in credit markets resulting from factors such as greater macroeconomic risk and significant deficiencies in the legal and regulatory environment.

1.7 Liquidity Risk

In this section, I study the effect of change in the liquidity risk (λ) on investment holdings in each country. First, I examine how the composition of foreign investment is affected by an increase in the liquidity risk in the host country.(comparative statics).Next, I introduce aggregate uncertainty about liquidity risk and analyze how the investment holding and prices are affected.

1.7.1 Comparative Statics

In this section, I analyze how the composition of foreign investment is affected by an increase in the liquidity risk in the host country.

Consider country A as a host country and country B as a source country. Suppose country A is in the type II pooling equilibria with respect to inward foreign investment, that is, it has inflows of both FDI and FPI. In this case, an increase in the liquidity risk in the host country (λ_A) leads to a lower level of total foreign investment. The effect on the composition of foreign investment is ambiguous and depends on the equilibrium. If economy is in the unique equilibrium then an increase in λ_A leads to more FDI and less FPI. However, if there are multiple equilibria then FDI may increase or decrease depending on the equilibrium.

As the liquidity risk increases, two effects take place. First, market liquidity is reduced reflecting the higher preference for safe liquid asset. This leads to lower level of foreign investment including FDI. At the same time, it reduces the adverse selection problem associated with direct investments: the fraction of FDI “lemons” is lower. This results in a smaller information discount on FDI, and therefore, leads

to a higher level of FDI.

If there are multiple equilibria and the economy is in the equilibrium with a larger fraction of direct investors (δ_{BA}) or if the equilibrium is unique, then the second effect dominates and an increase in liquidity risk in the host country leads to a higher level of FDI. If the economy is in the equilibrium with a smaller fraction of direct investors (δ_{BA}) then the first effect dominates and, therefore, an increase in liquidity risk in the host country leads to a lower level of FDI. In this case, the outflow of FDI is associated with self-fulfilling expectations: if an agent expects less agents to hold direct investments, then he chooses not to hold direct investment himself.

A similar argument applies to the case when country B is a host country. These results are summarized in the Proposition 2.

Proposition 2 *Suppose country $k \in \{A, B\}$ is in type II pooling equilibrium with respect to inward foreign investment. Then (i) if there is a unique equilibrium then an increase in liquidity risk results in a higher level of FDI; (ii) if there are multiple equilibria then an increase in liquidity risk results in a higher level of FDI in one equilibrium, and a lower level of FDI in another.*

1.7.2 Aggregate Uncertainty about Liquidity Risk

Suppose there are two aggregate liquidity states ($\lambda_{kL}, \lambda_{kH}$) for the host country such that $\lambda_{kL} < \lambda_{kH}$. The state λ_{kH} is a crisis state where the fraction of investors hit by a liquidity shock is larger. These states are realized with ex-ante probabilities $(1 - q)$ and q . Consider again country A as a host country, then the model by a

normal state $S_L = (\lambda_{AL}, \lambda_B)$ and a crisis state $S_H = (\lambda_{AH}, \lambda_B)$ where $\lambda_{AH} > \lambda_{AL}$.

All investment decisions at $t = 0$, such as fractions of direct investors $(\delta_{Ak}, \delta_{Bk})$ and direct and portfolio investment holdings $(x_{dk}^A, x_{pk}^A, x_{dk}^B, x_{pk}^B)$, are made before liquidity state S is realized. However, it affects the prices and demands for direct and portfolio investments in period one depend on which state is realized.

There are two ways in which the prices are affected, one is through the market liquidity and another is through the adverse selection problem associated with direct investment. The first effect is the dry up of market liquidity as more investors have to sell their asset holdings, and fewer investors are buying. Therefore, investment prices fall in order to clear the market. At the same time, direct investments are more likely to be sold before maturity due to a liquidity shock rather than because of the adverse productivity signal. Therefore, the market belief about the probability of receiving a liquidity shock (λ_d) is higher than in a crisis state relative to a normal state. This reduces the adverse selection problem and results in a smaller information discount on direct investment.

Then the depressed prices together with the reduced discount on direct investment capture the phenomenon of fire-sale FDI. The lower prices reflect the difficulty of finding buyers during the crisis. Aguiar and Gopinath [4] show that during the Asian financial crisis in late 1990s the median ratio of offer price to book value substantially declined. The low liquidity of domestic investors led to the significant increase in acquisitions involving foreign investors.

Next suppose that probability of a crisis depends on the previously realized state. So that conditional probability of transition from a normal state to a crises state is smaller than the conditional probability of remaining in a crisis state. The

transition matrix is given by $\begin{bmatrix} 1 - q_{LH} & q_{LH} \\ 1 - q_{HH} & q_{HH} \end{bmatrix}$ where $q_{HH} > q_{LH}$.

Then we can compare the equilibria sequentially, and analyze how the composition of foreign investment depends on the expectation of a liquidity crisis. Similarly to the comparative statics with respect to an increase in liquidity risk, a higher expectation of a liquidity crisis has two effects. One is reduced market liquidity since investors' preferences for liquidity are higher. Another is a smaller information discount on the prematurely sold direct investment. The first effect leads to less FDI while the second effect results in more FDI. If there is a unique equilibrium, then second effect (reduced adverse selection) dominates so higher liquidity risk leads to a higher level of FDI. If, however, there are multiple equilibria, higher liquidity risk may lead to a lower level of FDI.

Figure 1.6 illustrates the effect of an increase in probability of a liquidity crisis q on foreign direct and portfolio investment.

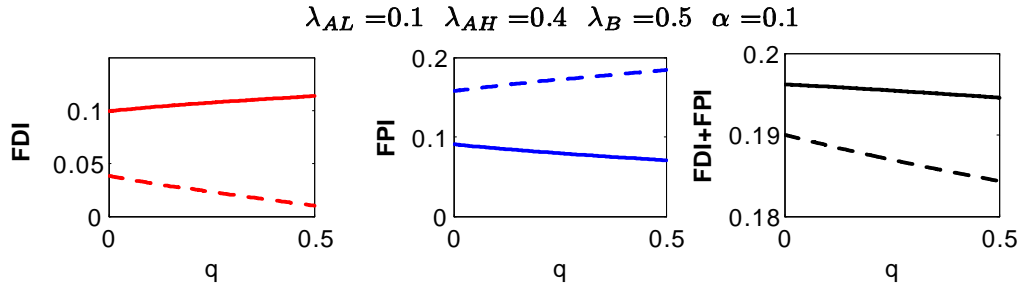


Figure 1.6: FDI_{BA} and FPI_{BA} as functions of probability of the crisis state s .

The results can be related to the empirically observed pattern of FDI during liquidity crises, as discussed in the following section.

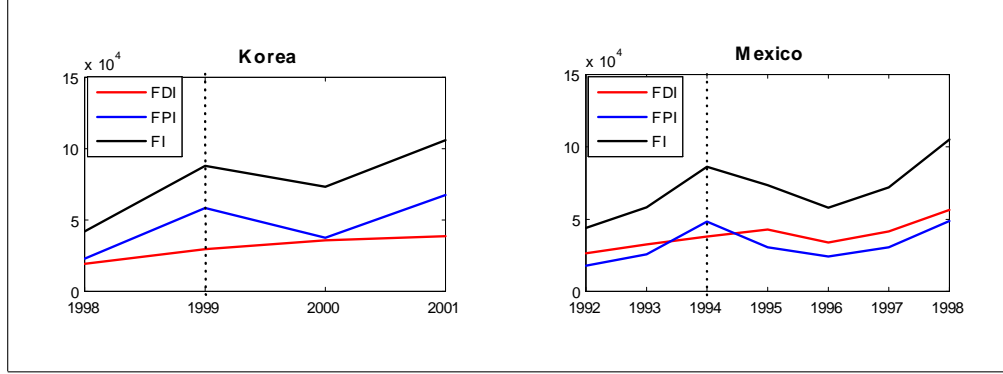


Figure 1.7: Crises in Korea and Mexico: inflow of FDI and outflow of FPI (millions of 2006 U.S. dollars).

1.8 Empirical Evidence

In this section, I consider the empirical data on foreign investment during the episodes of liquidity crises. The capital flows data is from the Lane and Milesi-Ferretti (2006) dataset.²⁴

On one hand, the positive effect of a higher liquidity risk on the inward FDI is consistent with the evidence of fire-sale FDI. Figure 1.7 shows the FDI and FPI flows into South Korea and Mexico in the time period around their respective financial crises in late 1990s and 1994.²⁵

Both Korea and Mexico can be viewed as a country B (a country with higher liquidity risk) in my model, and the financial crises can be interpreted as the increase in liquidity risk λ_B . Then, according to my model, if a country is in type I equilibria with respect to inward foreign investment, then the higher liquidity risk

²⁴They construct estimates of external assets and liabilities, distinguishing between foreign direct investment, portfolio equity investment, official reserves, and external debt for over 140 countries over the period of 1970-2004.

²⁵The East Asian financial crisis started in Thailand with the financial collapse of the Thai baht in 1997. Indonesia, South Korea, Malaysia, and the Philippines were the most affected by the crisis.

The Mexican (Tequila) crisis was triggered by the sudden devaluation of the Mexican peso in December, 1994.

leads to more FDI and less FPI. If a country is in type II equilibria with respect to inward foreign investment, then an increase in liquidity risk results in a higher level of FDI in one of the equilibria. As we can see from the figure, in Korea during the late 1990s crisis and in Mexico following the 1994 crisis the FDI level has been increasing while FPI level has declined.

Furthermore, the insurgence of FDI during liquidity crises is supported by empirical evidence on mergers and acquisitions in crises-stricken countries. Analyzing firm-level dataset on mergers and acquisitions in countries that underwent the Asian financial crises in late 1990s, Aguiar and Gopinath [4] find that during the crisis foreign acquisitions increased by 91% while domestic acquisitions declined by 27%. Moreover, Acharya, Shin, and Yorulmazer [1] observe that FDI inflows during financial crises are associated with acquisitions of stakes that grant control and, furthermore, the assets acquired in fire sales are subsequently re-sold quickly (flipped) to domestic buyers once the crisis has past.

On the other hand, my model provides a possibility of a decrease in FDI through self-fulfilling expectations. This possibility is consistent with the behavior of FDI during the early 1990s crisis in Sweden²⁶ and the 2001 crisis in Argentina²⁷. As Figure 1.8 shows, FDI has declined in both countries.

Sweden can be viewed as a country A . Suppose it is in the type II pooling equilibria with respect to inward foreign investment, that is, it has inflows of both FDI and FPI. If there are multiple equilibria then an increase in λ_A may lead to less FDI and more FPI depending on the equilibrium.

Argentina can be viewed as a country B . If it is in type II equilibria with

²⁶The Exchange Rate Mechanism crisis in Scandinavia in early 1990s.

²⁷Argentina defaults in December 2001.

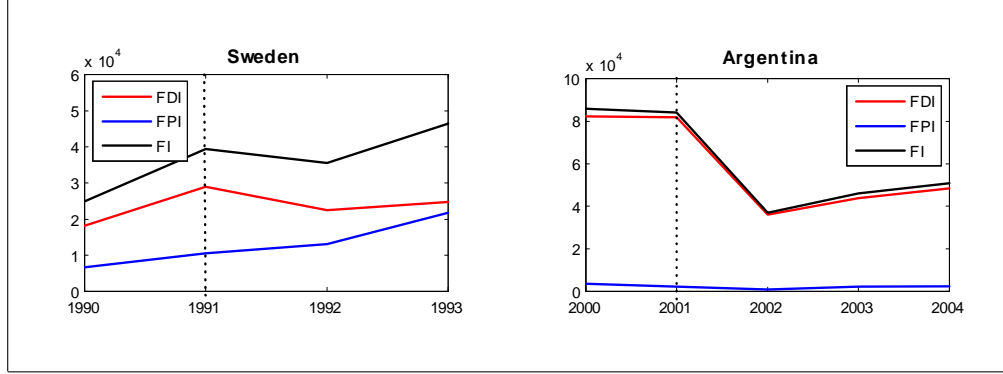


Figure 1.8: Crises in Sweden and Argentina: outflow of FDI (millions of 2006 U.S. dollars).

respect to inward foreign investment, then it has inflows only of FDI. Then an increase in the country liquidity risk λ_B may result in a lower level of FDI in one of the equilibria. The level of FPI into Argentina in early 2000s is almost at zero.

1.9 Conclusion

I analyze the composition of foreign investment between two countries which may differ in two dimensions: liquidity risk (probability of a liquidity crisis) and the investment productivity (fundamentals). I find that the country with a higher liquidity risk attracts less foreign investment, but a higher share of it is in the form of FDI. Also, a country with a larger uncertainty about investment productivity attracts more FDI relative to FPI since the marginal benefits from private information are larger. This is consistent with the empirical findings that countries that are less financially developed attract more capital in the form of FDI. This offers an explanation based on the difference in liquidity risk for the phenomenon of bilateral FDI flows among developed countries and one-way FDI flows from developed to emerging countries.

The effect on FDI of an increase in liquidity risk in the host country is ambiguous. If the economy is in the unique equilibrium then a higher liquidity risk leads to larger FDI holdings and smaller FPI holdings. This result is in line with the fire-sale FDI phenomenon. If, however, there are multiple equilibria then a higher liquidity risk may lead to the opposite effect: FDI declines. In this case, an outflow of FDI is induced by self-fulfilling expectations. This ambiguous impact of increased liquidity risk on foreign investment corresponds to the empirical evidence on capital flows during liquidity crises.

CHAPTER 2

THE ROLE OF ADVERSE SELECTION AND LIQUIDITY IN FINANCIAL CRISIS

2.1 Introduction

In the current crisis of 2007-2008, the market for securities backed by subprime mortgages was the first to suffer the sudden dry up in liquidity. Some of the possible explanations for illiquidity were the lack of transparency and the information asymmetries about the true value of the assets. In particular, the difficulty in assessing the fundamental value of the security may lead to the adverse selection.

Banks created structured financial products referred to as collateralized debt obligations (CDOs). CDOs were formed from diversified portfolios of mortgages and other types of assets such as corporate bonds, credit cards and auto loans. The pooled portfolios were sliced into different tranches which have been prioritized in how they absorb losses from the underlying portfolio. The top tranches were constructed to receive an AAA rating, these tranches were the first to be paid out of the cash flows and were widely considered to be safe. The most junior "equity" tranche (also referred to as "toxic waste") were to be paid out only after all other tranches have been paid. The junior tranches were usually held by the issuing bank; they were traded infrequently and were therefore hard to value.¹ Also, these structured finance products received overly optimistic ratings from the credit rating agencies. One of the reasons the underlying securities default risks were underestimated is that the statistical models were based on the historically low mortgage default and delinquency rates.(Brunnermeier [16])

¹Brunnermeier [16]

CDOs written on subprime mortgages had skewed payoffs: they offered high expected return in most states of nature but suffered catastrophic losses in extremely bad states. When economy is in a normal state with strong fundamentals, the asymmetric information does not significantly affect the value of mortgage backed securities (MBS). However, when an economy is subject to a negative shock, the value of the security becomes more sensitive to private information and the adverse selection may influence the trading decisions. (Morris and Shin [67]) When in February 2007 subprime mortgage defaults had increased,² a large fraction of CDOs were downgraded³. The impact of declining housing prices on MBS depended on the exact composition of mortgages that backed the securities. Due to the complexity of structured financial products and heterogeneity of the underlying asset pool, the owners have an informational advantage in estimating how much those securities are worth. This asymmetric information about the true value of the asset generates the lemons problem⁴: a buyer does not know whether the seller is selling the security because of a sudden need for liquidity, or because the seller is trying to get rid of the toxic assets. This adverse selection issue can lead to the market illiquidity reflecting buyers' beliefs that most securities offered for sale are of low quality.

The flight to liquidity can amplify the effect of adverse selection during the crisis leading to the increased asset price volatility and possibly to a complete breakdown of trading. As market liquidity falls, it becomes difficult to find trading partners which leads to a fire-sale pricing.⁵ The deleveraging that accompanies the initial

²This increase in subprime mortgage defaults triggered the liquidity crisis in February 2007. (Brunnermeier [16])

³"27 of the 30 tranches of asset-backed collateralized debt obligations underwritten by Merrill Lynch in 2007, saw their triple-A ratings downgraded to "junk" Overall, in 2007, Moody's downgraded 31 percent of all tranches for asset-backed collateralized debt obligations it had rated and 14 percent of those initially rated AAA." (Coval, J. Jurek, and E. Stafford [25])

⁴Akerlof (1970)

⁵The haircut on ABSs increased from 3-5% in August 2007 to 50-60% in August 2008. The

shock can further aggravate the adverse selection problem.⁶ Because of the losses on their MBS, some banks became undercapitalized; however, their attempts to recapitalize push their market price further down. This reflects the investors' fear that any bank that issues new equity or debt may be overvalued, leading to the liquidity crunch.

In this paper, I develop a model that illustrates how adverse selection in an asset market can lead to an equilibrium with no trade during the crisis. Also, I analyze the role of market liquidity and the role of expectations in amplifying the effect of adverse selection.

In my model, agents have the Diamond-Dybvig⁷ type of preferences: they consume in period one or in period two, depending on whether they receive a liquidity shock in period one. In period zero, investors choose how much to invest into risky long-term assets which have idiosyncratic payoffs. In period one, liquidity shocks are realized and, subsequently, risky investments are traded in the financial market. The late consumers (who have not experienced a liquidity shock) are the buyers in the financial market.

I begin by examining the portfolio choice when investors have private information about their investment payoff and it is public information which investors have received a liquidity shock. Then I analyze the situation when the identity of investors hit by a liquidity shock is private information. In the latter case, investors can take advantage of their private information by selling the low-payoff

haircut on equities increased from 15% to 20% for the same period. (Gorton and Metrick (2009))

⁶"The large haircuts on some securities could be seen as a response by leveraged entities to the potential drying up of trading possibilities in the asset-backed securities (ABS) market. The equity market, in contrast, is populated mainly with non-leveraged entities such as mutual funds, pension funds, insurance companies and households, and hence is less vulnerable to the drying up of trading partners." Morris and Shin [67]

⁷Diamond and Dybvig (1983)

investments and keeping the ones with high payoffs. This leads to the lemons problem. If market is liquid then informed investors can gain from trading on private information by pretending to be liquidity traders (investors who experienced a liquidity shock). However, if the fraction of low quality assets offered for sale is large then the adverse selection can lead to the market illiquidity.

Following the Allen and Gale "cash-in-the-market" framework⁸, in my model liquidity depends on the amount of the safe asset held by the investors that is available to buy risky assets from liquidity traders. The market liquidity (demand for risky investments in the interim period) depends on the investors liquidity preference. Allen and Gale [8] show that the "cash-in-the-market" pricing leads to the market prices below fundamentals if the preference for liquidity is high. I demonstrate that the presence of adverse selection in the market can further depress the market prices exacerbating the asset price volatility.

I show that if a crisis is accompanied by the flight to liquidity, the effect of adverse selection can be amplified leading to the fire-sale pricing or a breakdown of trade during the crisis. Furthermore, I show that underestimating the likelihood of the crisis can aggravate the adverse selection effect as well. Next, I analyze the investment choice from the central planner prospective. The central planner can improve upon the market allocation by eliminating the lemons problem.

⁸The amount of cash in the market depends on the participants liquidity preference. The higher the average liquidity preference of investors in the market, the greater is the average level of the safe assets in portfolios and the greater is the market ability to absorb liquidity trading without large price changes. (Allen and Gale [7])

2.2 Model

I consider a model with three dates indexed by $t = 0, 1, 2$. There is a continuum of ex-ante identical agents with an aggregate Lebesgue measure of unity. There is only one good in the economy that can be used for consumption and investment. All agents are endowed with one unit of good at date $t = 0$, and nothing at the later dates.

2.2.1 Preferences

Agents consume at date one or two, depending on whether they receive a liquidity shock at date one. The probability of receiving a liquidity shock in period one is denoted by λ . So λ is also a fraction of investors hit by a liquidity shock. Investors who receive a liquidity shock have to liquidate their risky long-term asset holdings and consume all their wealth in period one. So they are effectively early consumers who value consumption only at date $t = 1$. I will also refer to them as liquidity traders. The rest are the late consumers who value the consumption only at date $t = 2$.

Investors have Diamond-Dybvig type of preferences:

$$U(c_1, c_2) = \lambda u(c_1) + (1 - \lambda)u(c_2) \tag{2.1}$$

where c_t is the consumption at dates $t = 1, 2$. In each period, investors have logarithmic utility: $u(c_t) = \log c_t$.

2.2.2 Investment Technology

Agents have access to two types of constant returns investment technologies. One is a storage technology (also called the safe asset or cash), which has zero net return: one unit of safe asset pays out one unit of safe asset in the next period. Another type of technology is a long-term risky investment project (also called a risky asset). In period two, the risky investment in project i has a random idiosyncratic payoff R^i per unit of investment. Each investor i has a choice of starting his own investment project i by investing a fraction of his endowment. The investor can start only one project, and each project has only one owner; the payoff realizations are independent across investments.

The payoff of each investment i is an independent realization of a random variable R^i that takes two values: a low value R_L with probability π_s and a high value R_H with probability $(1 - \pi_s)$ where $s \in \{1, 2\}$. There are two states of nature $s = 1$ and $s = 2$ that are revealed at $t = 1$. The state 1 is a normal state where the fraction of low quality assets is small: $\pi = \pi_1$. The state 2 is a crisis state with a significantly larger fraction of low quality assets: $\pi = \pi_2 > \pi_1$. Also, π_s is a fraction of investments with low payoff in state s . These states are realized with ex-ante probabilities $(1 - q)$ and q . I will also use the notation $q_1 = 1 - q$ and $q_2 = q$.

The expected payoff of each individual risky project in state s is denoted by $\bar{R}_s = \pi_s R_L + (1 - \pi_s) R_H$ with $R_L < 1 < R_H$. The expected payoff is denoted by $\bar{R} = (1 - q) \bar{R}_1 + q \bar{R}_2$ with $\bar{R} > 1$. The long-term asset can be liquidated prematurely at date $t = 1$, in this case, one unit of the risky asset yields r units of the good, where $R_L < r < 1$. The holdings of the two-period risky asset can be traded in financial market at date $t = 1$. Table 2.1 summarizes the payoff

Table 2.1: Payoff structure.

time	0	1	2
safe asset	$1 \rightarrow$	$1 \rightarrow$	1
investment	$1 \rightarrow$	$r \rightarrow$	R^i

structure.

The following assumptions on asset returns parameters are maintained throughout.

Assumption 1. $R_L < 1 < R_H$.

Assumption 2. $(1 - q) \bar{R}_1 + q \bar{R}_2 > 1$

This assumption ensures that a risky investment is always more productive than the safe asset.

Assumption 3. $r \geq \underline{r} = \sum_{s=1,2} q_s \frac{R_L((1-\lambda)R_H + \lambda \bar{R}_s) + R_H(1-\lambda)(1-\pi_s)(R_H - R_L)}{((1-\lambda)R_H + \lambda \bar{R}_s + (1-\lambda)(1-\pi_s)(R_H - R_L))}$

This assumption rules out the situation when a risky asset dominates the safe asset at $t = 1$. If $r < \underline{r}$ then the market price at $t = 1$ is greater than one, therefore, no one will choose to hold the safe asset at $t = 0$. In particular, this assumption implies that $r > R_L$.

Assumption 4. $r \leq \bar{r} : EU(p(\bar{r}), x(\bar{r})) \geq (1 - \lambda) \sum_{s=1,2} q_s \log(\bar{R}_s / p(\bar{r}))$

This assumption rules out the situation when the safe asset dominates a risky asset at $t = 1$. If $r > \bar{r}$ then the return on the risky asset bought at $t = 1$ is higher than the return on investment made at $t = 0$, so no one will choose to invest in risky projects at $t = 0$. In particular, this assumption implies that $r < 1$.

2.2.3 Information

At date $t = 0$, investors make investment choices between the two technologies, safe and risky, in proportion x and $(1 - x)$ respectively. They choose their asset holdings to maximize their expected utility.

At date $t = 1$, the liquidity shocks and the aggregate state are realized, and the financial market opens. If investors have not received a liquidity shock, they privately observe the payoff of investment they own. The supply of the risky asset comes from the investors who have experienced a liquidity shock. The demand for risky asset comes from investors who have not received a liquidity shock.

I will consider two cases. In the first case, it is public information which investor have experienced a liquidity shock. If an investor gets a liquidity shock, he sells or liquidates his holdings of the risky asset in order to consume as much as possible in period one. If an investor is not hit by a liquidity shock and learns that his investment has low payoff, he can liquidate it, receiving r units of the good per unit of investment.

In the second case, the identity of investors hit by a liquidity shock is private information. Therefore, after observing investment payoffs, agents can take advantage of this private information by selling low quality projects in the market at date $t=1$. In this case, buyers are not able to distinguish whether an investor is selling his asset holdings because of its low payoff or because of the liquidity needs. This generates adverse selection problem, and leads to the discount on the investments sold before maturity.

2.3 Equilibrium

2.3.1 Equilibrium without Adverse Selection

First, I consider the case where the identity of investors hit by a liquidity shock is public information. Therefore, there is no adverse selection. All risky assets at $t=1$ are sold by liquidity traders who cannot wait for the maturity of their investments at date $t = 2$.

Since all the investments have idiosyncratic payoffs, the expected payoff of the risky asset sold in period one is \bar{R}_s in state s . All risky assets sold at $t = 1$ are aggregated in the market, hence, the variance of the asset bought at date $t = 1$ is zero. Therefore, the return on risky asset bought in period one is \bar{R}_s/p_s , where p_s is the market price in state s . The late consumers will be willing to buy risky asset at date $t = 1$ if the market price p_s is less than the expected payoff \bar{R}_s . The earlier consumers will be willing to sell their projects if the market price p_s is greater than the liquidation value r .

At date $t = 0$, investors choose the investment allocations between the risky and safe technologies, in proportion x and $(1-x)$ respectively, in order to maximize

their expected utility.

$$\begin{aligned}
& \lambda \log c_1 + (1 - \lambda) \sum_{s=1,2} q_s (\pi_s \log c_{2L}(s) + (1 - \pi_s) \log c_{2H}(s)) \quad (2.2) \\
s.t. \quad (i) \quad & c_1(s) = \begin{cases} 1 - x + p_s x & \text{if } p_s \geq r \\ 1 - x + r x & \text{if } p_s < r \end{cases} \\
(ii) \quad & c_{2H}(s) = \begin{cases} x R_H + y_s \bar{R}_s & \text{if } p_s \geq r \\ x R_H + (1 - x) & \text{if } p_s < r \end{cases} \\
(iii) \quad & c_{2L}(s) = \begin{cases} x r + y_s \bar{R}_s & \text{if } p_s \geq r \\ x r + (1 - x) & \text{if } p_s < r \end{cases}
\end{aligned}$$

The consumption of early consumers in state s is denoted by $c_1(s)$ and the consumption of late consumers in state s is denoted by $c_{2j}(s)$ where $j = L, H$ refers to payoff of an investment project i .

The late consumers will be willing to buy risky assets at $t = 1$ if the market price p is less than the expected payoff \bar{R} . Therefore, the demand for risky asset at $t = 1$ in state s is given by

$$y(s) = \begin{cases} \frac{1-x}{p_s} & \text{if } p_s \leq \bar{R}_s \\ 0 & \text{if } p_s > \bar{R}_s \end{cases} \quad (2.3)$$

Therefore, the aggregate demand at $t = 1$ in state s is given by

$$D(s) = \begin{cases} (1 - \lambda) \frac{1-x}{p_s} & \text{if } p_s \leq \bar{R}_s \\ 0 & \text{if } p_s > \bar{R}_s \end{cases} \quad (2.4)$$

The earlier consumers will be willing to sell their projects if the market price p is greater than the liquidation value r . Therefore, the aggregate supply at $t = 1$

in state s is given by

$$S(s) = \begin{cases} \lambda x & \text{if } p_s \geq r \\ 0 & \text{if } p_s < r \end{cases} \quad (2.5)$$

The price in state is determined by the market clearing conditions:

$$\lambda x p_s = (1 - \lambda)(1 - x)$$

Since the investment allocations are determined at $t = 0$ and there are no aggregate uncertainty about the probability of a liquidity shock λ , the price of the risky asset sold at $t=1$ is the same in both states: $p_1 = p_2 \equiv p$ where

$$p = \frac{(1 - \lambda)(1 - x)}{\lambda} \frac{1}{x}$$

Proposition 3 *If assumption 1-4 are satisfied, then there exists a unique equilibrium, and the equilibrium allocation into long-term risky investment x and the market price of investment sold at date one p are given by*

$$p = \frac{\lambda + \sum_{s=1,2} q_s \left(\pi_s \frac{\bar{R}_s}{r + \bar{R}_s \frac{\lambda}{(1-\lambda)}} + (1 - \pi_s) \frac{\bar{R}_s}{R_H + \bar{R}_s \frac{\lambda}{(1-\lambda)}} \right)}{\lambda + \sum_{s=1,2} q_s \left(\pi_s \frac{r}{r + \bar{R}_s \frac{\lambda}{(1-\lambda)}} + (1 - \pi_s) \frac{R_H}{R_H + \bar{R}_s \frac{\lambda}{(1-\lambda)}} \right)} \quad (2.6)$$

$$x = \begin{cases} (1 - \lambda) \left(\lambda + \sum_{s=1,2} q_s \left(\pi_s \frac{r}{r + \bar{R}_s \frac{\lambda}{(1-\lambda)}} + (1 - \pi_s) \frac{R_H}{R_H + \bar{R}_s \frac{\lambda}{(1-\lambda)}} \right) \right) & \text{if } p \geq r \\ (1 - \lambda) \left(1 - \sum_{s=1,2} q_s \pi_s \right) \frac{1}{(1-r)} + \left(\lambda + (1 - \lambda) \sum_{s=1,2} q_s \pi_s \right) \frac{1}{(1-R_H)} & \text{if } p < r \end{cases} \quad (2.7)$$

Furthermore, the investment allocation and welfare are larger in the market equilibrium (when $p \geq r$) relative to an equilibrium with no trade (when $p < r$).

The equilibrium consumption of early consumers is the same in both states and is given by:

$$c_1 = \begin{cases} \left(\lambda + (1 - \lambda) \sum_{s=1,2} q_s \left(\pi_s \frac{\bar{R}_s}{(1-\lambda)r + \lambda \bar{R}_s} + (1 - \pi_s) \frac{\bar{R}_s}{(1-\lambda)R_H + \lambda \bar{R}_s} \right) \right) & \text{if } p \geq r \\ \left(\lambda + (1 - \lambda) \sum_{s=1,2} q_s \pi_s \right) \frac{R_H - r}{R_H - 1} & \text{if } p < r \end{cases} \quad (2.8)$$

The consumption of late consumers with low payoff investment in state s is given by

$$c_{2L}(s) = \begin{cases} (1 - \lambda) \left(r + \frac{\lambda}{1 - \lambda} \bar{R}_s \right) & \text{if } p \geq r \\ \left(\lambda + \sum_{s=1,2} q_s \left(\pi_s \frac{r}{r + \bar{R}_s \frac{\lambda}{1 - \lambda}} + (1 - \pi_s) \frac{R_H}{R_H + \bar{R}_s \frac{\lambda}{1 - \lambda}} \right) \right) & \text{if } p < r \\ \left(\lambda + (1 - \lambda) \sum_{s=1,2} q_s \pi_s \right) \frac{R_H - r}{R_H - 1} & \text{if } p < r \end{cases} \quad (2.9)$$

The consumption of late consumers with high payoff investment in state s is given by

$$c_{2H}(s) = \begin{cases} (1 - \lambda) \left(R_H + \frac{\lambda}{1 - \lambda} \bar{R}_s \right) & \text{if } p \geq r \\ \left(\lambda + \sum_{s=1,2} q_s \left(\pi_s \frac{r}{r + \bar{R}_s \frac{\lambda}{1 - \lambda}} + (1 - \pi_s) \frac{R_H}{R_H + \bar{R}_s \frac{\lambda}{1 - \lambda}} \right) \right) & \text{if } p < r \\ (1 - \lambda) \left(1 - \sum_{s=1,2} q_s \pi_s \right) \frac{R_H - r}{1 - r} & \text{if } p < r \end{cases} \quad (2.10)$$

2.3.2 Equilibrium with Adverse Selection

Now suppose the identity of investors who have received a liquidity shock is private information. Therefore, after observing investment payoff, agents can take advantage of this private information by selling low productive investments in the market at date $t=1$. This generates the adverse selection problem and therefore, leads to the discount on the price of risky assets sold at $t = 1$. Investors always can choose to liquidate the project if it yield a low payoff.

The investor who buys a risky asset at date $t = 1$, does not know whether it is sold due to the liquidity shock or because of its low payoff. The buyers believe that with probability λ investment is sold due to a liquidity shock, and with probability $(1 - \lambda)(1 - \pi_s)$ it sold because of the low payoff. Hence, buyers believe that the payoff of the prematurely sold risky assets in state s is \tilde{R}_s such that

$$\tilde{R}_s = \frac{\lambda}{\lambda + (1 - \lambda)\pi_s} \bar{R}_s + \frac{(1 - \lambda)\pi_s}{\lambda + (1 - \lambda)\pi_s} R_L \quad (2.11)$$

The late consumers will be willing to buy risky asset at $t=1$ if the market price p is less than the expected payoff \tilde{R} . Therefore, the demand for risky asset at $t = 1$ is given by

$$y_s = \begin{cases} \frac{1-x}{p_s} & \text{if } p_s < \tilde{R}_s \\ 0 & \text{if } p_s > \tilde{R}_s \end{cases} \quad (2.12)$$

The earlier consumers will be willing to sell their projects if the market price p_s is greater than the liquidation value r .

The price in state s is determined by market clearing conditions:

$$(\lambda + (1 - \lambda)\pi_s)xp_s = (1 - \lambda)(1 - x) \quad (2.13)$$

Therefore, the market price in state s can be expressed as

$$p_s = \frac{(1 - \lambda)}{(\lambda + (1 - \lambda) \pi_s)} \frac{(1 - x)}{x} \quad (2.14)$$

Note, that the price is no longer the same in both states since the fraction of low productive investments is larger in a crisis state: $\pi_2 > \pi_1$. Therefore, the price in the crisis state is lower than the price in the normal state: $p_2 < p_1$.

Investors choose their asset holdings $(x, 1 - x)$ to maximize their expected utility:

$$\begin{aligned} & \lambda \log c_1 + (1 - \lambda) \sum_{s=1,2} q_s (\pi_s \log c_{2L}(s) + (1 - \pi_s) \log c_{2H}(s)) \quad (2.15) \\ \text{s.t. } & (i) \quad c_1(s) = \begin{cases} 1 - x + p_s x & \text{if } p_s \geq r \\ 1 - x + r x & \text{if } p_s < r \end{cases} \\ & (ii) \quad c_{2H}(s) = \begin{cases} x R_H + (1 - x) \tilde{R}_s / p_s & \text{if } p_s \geq r \\ x R_H + (1 - x) & \text{if } p_s < r \end{cases} \\ & (iii) \quad c_{2L}(s) = \begin{cases} x p + (1 - x) \tilde{R}_s / p_s & \text{if } p_s \geq r \\ x r + (1 - x) & \text{if } p_s < r \end{cases} \end{aligned}$$

Proposition 4 *If assumptions 1-4 are satisfied then there exists a unique equilibrium. There are three possible equilibrium types: I. equilibrium with market trading in both states; II. equilibrium with market trading in normal state $s = 1$ and no trade in a crisis state $s = 2$; III. equilibrium with no trade in both states. Furthermore, the presence of adverse selection leads to a lower level of investments x , and lower welfare relative to an equilibrium without adverse selection.*

The presence of adverse selection leads to the lower price level and price volatil-

ity across states. The market price in a crisis state is lower relative to the normal state since the fraction of low quality assets is larger. As a result, assets offered for sale at $t = 1$ have lower expected return. Informed investors are benefiting from the private information at the expense of liquidity traders. Furthermore, adverse selection leads to the loss in aggregate welfare since informed investors sell low productive investments instead of liquidating them.

Properties of Equilibrium

Probability of a crisis state. The probability of a crisis state q reflects the investors' beliefs about the likelihood of a crisis. In this section, I examine how the equilibrium changes with respect to changes in q .

Corollary 1. *If investors believe a crisis state is more likely to occur (q is larger) then (i) investment allocation is smaller; (ii) market prices are higher; (iii) expected utility is lower. If the economy is in a type II equilibrium with market trading in normal state and no trade in a crisis state then increase in q may lead to shift a type I equilibrium with market trading in both states.*

The higher probability of a crisis state q implies a higher probability of the asset becoming a lemon, which makes asset ex-ante less less profitable. Therefore, the increase in probability of a crisis state q leads to a lower level of investment allocation and lower expected utility. The smaller investment at $t = 0$ implies the smaller supply and larger demand for risky assets at $t = 1$. This leads to higher market prices (in both type I and II equilibria).

The fact that the market price is increasing in the probability of a crisis state q makes it is possibility to move from one type of equilibrium to another. Suppose an

economy is in type II equilibrium where there is no trade in a crisis state. Suppose the probability of a crisis state q increased, i.e., investors believe a crisis is now more likely to occur. Then it is possible that the price in a crisis state will increase sufficiently to switch to type I equilibrium with market trading in both states. (If an economy is initially in type I equilibrium then the type of the equilibrium will not change if q is increased. If an economy is in type II equilibrium and the probability q is decreased then the equilibrium type will not change either.)

Consider the following numerical example. The asset return parameters are given $R_L = 0$, $R_H = 1.3$, $r = 0.65$, the fraction of low quality investments in a normal state: $\pi_1 = 0.05$ and in a crisis state $\pi_2 = 0.25$, and probability of a liquidity shock $\lambda = 0.3$. In this example, 5% of assets become lemons (with zero payoff) in a normal state, and in a crisis state, the quarter of all assets are lemons. Figure 2.1 depicts the equilibrium values of investment, prices and expected utility as a function of probability of a crisis state q . At $q = 0.25$, there is a switch from an equilibrium with no trade in a crisis state to an equilibrium with trading in both state.

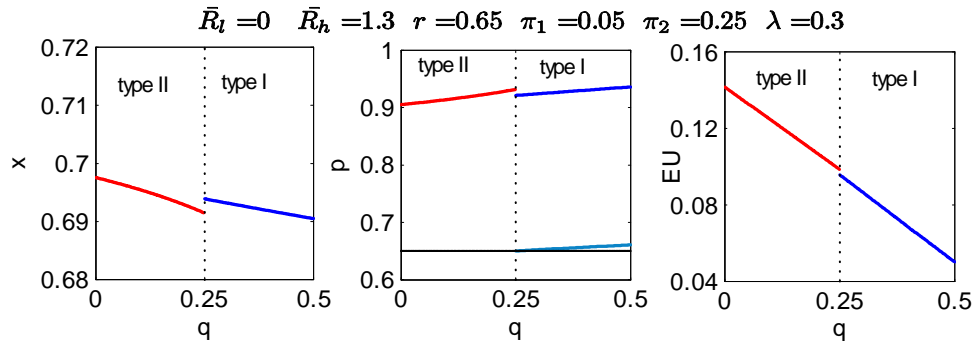


Figure 2.1: Changes in equilibrium values of investment, prices and expected utility as a function of probability of the crisis.

Therefore, the initial expectation can affect the type of equilibrium. The un-

derestimating the likelihood of a crisis may result in the no-trade outcome if a crisis state is realized.

Suppose a probability of a crisis q depends on the previously realized state. So that conditional probability of transition from a normal state to a crises state is smaller than the conditional probability of remaining in a crisis state. The transition matrix is given by $\begin{bmatrix} 1 - q_{12} & q_{12} \\ 1 - q_{22} & q_{22} \end{bmatrix}$ where $q_{22} > q_{12}$ and $q_{jk} = \Pr(s = s_k | s = s_j)$. Then we can compare equilibria sequentially.

Let's look again at the numerical example considered before. Suppose $q_{11} = 0.1$ and $q_{22} = 0.5$. If an economy is in a normal state then it is in type II equilibrium: if the crisis is realized, there is no trading. Once economy is in a crisis state, the beliefs are revised and investment allocation are adjusted, and an economy moves to the type I equilibrium. So, the market trading is resumed next period even if the crisis state persists.

Liquidity preference. Now consider the situation when a crises is accompanied by an exogenous increase in liquidity preference λ in addition to a larger fraction of low quality assets.

Corollary 2. *Suppose the economy is type I equilibrium with market trading in both states. The increase in liquidity preference λ in a crisis state may lead to shift a type II equilibrium with market trading in normal state and no trade in a crisis state.*

The price is a decreasing function of preference for liquidity λ . Therefore, the higher preference for liquidity λ in a crisis state results in the further decrease of the market price relative to a normal state. Hence, the lack of liquidity during the

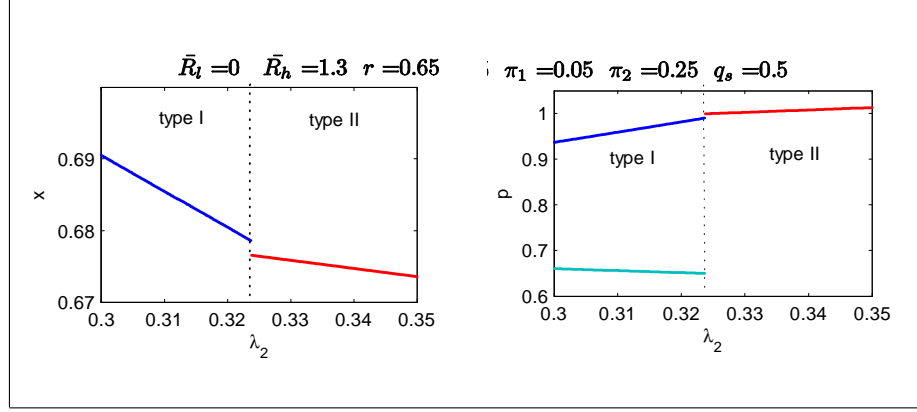


Figure 2.2: Changes in equilibrium values of investment and prices as a function of probability of the liquidity shock.

crisis may amplify the adverse selection problem pushing the asset prices further down and possibly leading to a complete breakdown of trade. This reflects the fire-sale phenomenon when depressed prices reflect the difficulty of finding buyers during the crisis.

Again consider the numerical example: $R_L = 0$, $R_H = 1.3$, $r = 0.65$, the fraction of low quality investments in a normal state: $\pi_1 = 0.05$ and in a crisis state $\pi_2 = 0.25$, and probability of a liquidity shock $\lambda = 0.3$. Figure 2.2 illustrates the effect of an increase in the liquidity preference in a crisis state λ_2 from 0.3 to 0.35 on the equilibrium investment and prices. When preference for liquidity is the same in both states $\lambda_1 = \lambda_2 = 0.3$, there is trading in both states. However, if $\lambda_2 > 0.325$ then there is no trade in a crisis state.

The next figure 2.3 depicts the equilibrium investment and prices as a function of probability of a crisis state q when the preference for liquidity in a crisis state is higher: $\lambda_1 = 0.3$ and $\lambda_2 = 0.31$. The threshold value of a crisis likelihood (where economy switches from type II to type I equilibrium) is larger relative to the case when the liquidity preference in both states are the same $\lambda_1 = \lambda_2 = 0.3$. If a

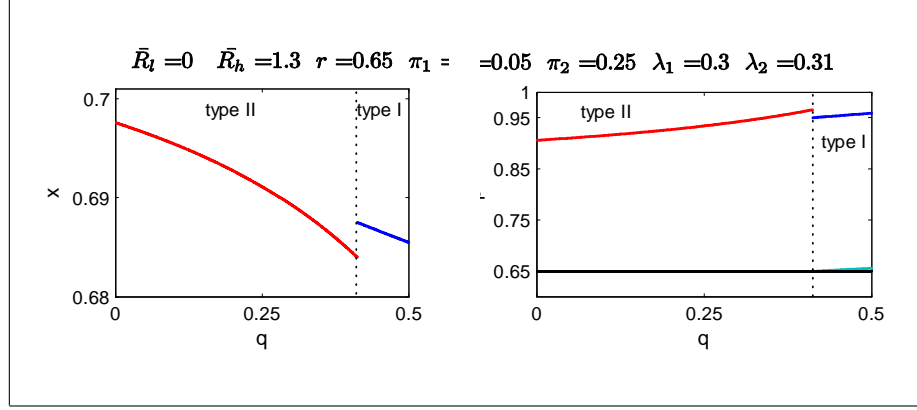


Figure 2.3: Changes in equilibrium values of investment and prices as a function of probability of the crisis.

crisis is accompanied by flight to liquidity, the adverse selection effect is magnified exacerbating the asset price volatility.

2.3.3 Central Planner Allocation

In this section, I analyze the equilibrium from the central planner prospective.

First, consider the case when it is public information which investor has received a liquidity shock. Then the central planner solve the following maximization problem:

$$\begin{aligned} \max_x & \{ \lambda \log c_1 + (1 - \lambda) \sum_{s=1,2} q_s (\pi_s \log c_{2L}(s) + (1 - \pi_s) \log c_{2H}(s)) \} \\ \text{s.t. } & (i) \quad c_1 = \frac{1-x}{\lambda} \\ & (ii) \quad c_{2L}(s) = x \left(r + \bar{R}_s \frac{\lambda}{1-\lambda} \right) \\ & (iii) \quad c_{2H}(s) = x \left(R_H + \bar{R}_s \frac{\lambda}{1-\lambda} \right) \end{aligned}$$

The optimal investment allocation is $x^o = (1 - \lambda)$ and the consumption allo-

cations are given by

$$\begin{aligned} c_1^o &= 1 \\ c_{2L}^o(s) &= (1 - \lambda)r + \lambda\bar{R}_s \\ c_{2H}^o(s) &= (1 - \lambda)R_H + \lambda\bar{R}_s \end{aligned}$$

Next, suppose that the identity of investors hit by a liquidity shock is private information. This adds incentive compatibility constraints to the maximization problem: the period one consumption c_1 has to be less than any of the consumptions in period two. The smallest period two consumption is attained in state 2 with low productive investment: $c_{2L}(s_2)$. Therefore,

$$(iv) \quad c_1 \leq c_{2L}(s_2)$$

If an equilibrium $(c_1^o, c_{2j}^o(s) : j = L, H)$ satisfies the incentive compatibility constraint (iv) then it remains an equilibrium.

If not, then the equilibrium investment allocation x^{oo} is given by

$$x^{oo} = \frac{1 - \lambda}{(1 - \lambda) + \lambda((1 - \lambda)r + \lambda\bar{R}_2)}$$

The consumption allocations are given by

$$\begin{aligned} c_1^{oo} &= c_{2L}^{oo}(s) = \frac{(1 - \lambda)r + \lambda\bar{R}_2}{(1 - \lambda) + \lambda((1 - \lambda)r + \lambda\bar{R}_2)} \\ c_{2H}^{oo}(s) &= \frac{(1 - \lambda)R_H + \lambda\bar{R}_2}{(1 - \lambda) + \lambda((1 - \lambda)r + \lambda\bar{R}_2)} \end{aligned}$$

Note, that the investment allocation in the new incentive compatible equilibrium is larger than in the previous one: $x^{oo} > x^o$. This benefits late consumers with high productive investments at the expense of early consumers and late consumers

with low productive investments. Furthermore, the second period consumption depends on which state is realized, however, it does not depend on the probability of a crisis.

Now we can compare the market vs the central planner equilibrium. First, let's look at the equilibrium without adverse selection. The investment allocation in a market equilibrium is larger than in the central planner solution. The expected consumption of late consumers is larger, and the expected consumption of early consumers is smaller than the corresponding central planner consumption allocation. (See figure 2.4 for an example.) The market equilibrium is optimal when $p = 1$.⁹

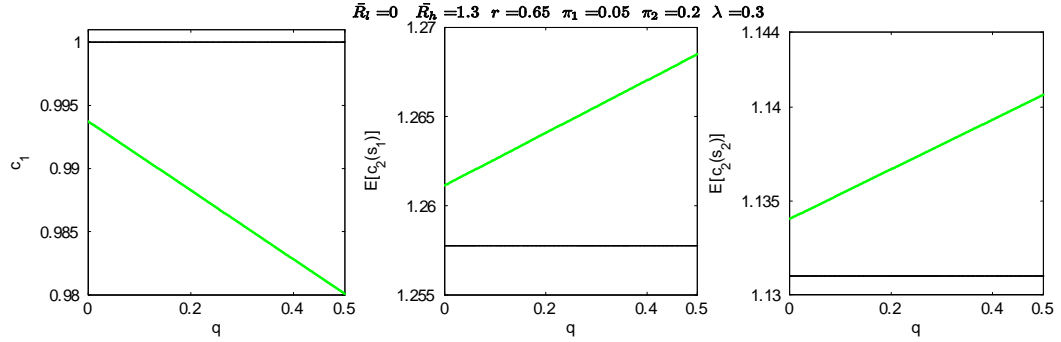


Figure 2.4: Consumption allocation in equilibrium without adverse selection as a function of probability of the crisis.

The adverse selection results in a lower consumption for both early and late consumers in each state. However, the late consumers with low productive investment benefit from adverse selection and get a higher level of consumption in a normal state relative to the central planner allocation. The rest of the investors consume less. The market equilibrium with adverse selection is not optimal. (See figure 2.5 for an example.) The central planner can improve upon the market equilibrium by preventing the adverse selection.

⁹See section A.3 of the Appendix for the proof.

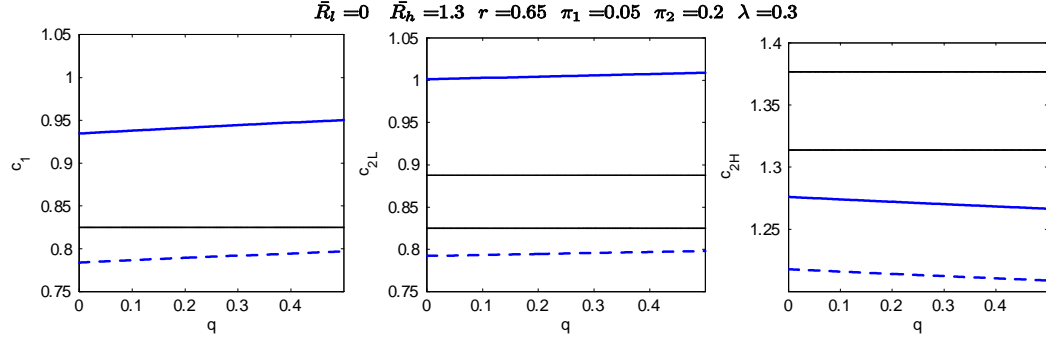


Figure 2.5: Consumption allocation in equilibrium with adverse selection as a function of probability of the crisis.

2.4 Conclusion

I analyze the effect of adverse selection in the asset market. The asymmetric information about asset returns generates the lemons problem when buyers do not know whether the asset is sold because of its low quality or because the seller's sudden need for liquidity. This adverse selection can lead to the market illiquidity reflecting the buyers' belief that most assets that are offered for sale are of low quality. The lack of market liquidity and underestimating the likelihood of a crisis can amplify the effect of adverse selection leading to the increased asset price volatility and possibly to a breakdown of trade during the crisis.

CHAPTER 3

EQUITY HOME BIAS UNDER AMBIGUITY AVERSION

3.1 Introduction

Equity home bias is a well known puzzle in international finance, referring to a wide disparity between the actual portfolio weights and the weights recommended by the international equity portfolio theory. Under ideal conditions, the international capital asset market model predicts that investors should hold equities from around the world in proportion to their market capitalization. However, according to the empirical findings of French and Poterba [36] and Tesar and Werner [74], investors hold a substantially larger proportion of their wealth in domestic assets: US investors hold 92.2% of their equity portfolio in domestic stocks; Japanese investors - 95.7%; UK investors - 92%; German investors - 79%; French investors - 89.4%, and Canadian investors - 93.4%. This observed high concentration in domestic equity has become known as "equity home bias".

There have been various attempts to explain this puzzle. The first approach is based on information asymmetries¹, hedging possibilities against domestic risk², and barriers to international investment such as restrictions on international capital flows³, withholding taxes, and transactions costs⁴. Another approach focuses on investors behavioral biases, e.g., optimism about their domestic markets⁵ and preference for the familiar⁶. Lewis [62] and Strong and Xin [72] provide an ex-

¹Gehrig (1993) [37], Kang and Stulz (1997) [50],Ahearne, Grierer, Warnock (2004)[5],Jeske (2001) [48]

²Baxter and Jermann(1995) [11], Cooper and Kaplanis(1994) [24], Uppal (1992) [76]

³Black (1974) [13], Stulz (1981) [73]

⁴Tesar and Werner (1995) [74], Warnock (2002) [79], Obstfeld (2000) [69]

⁵French and Poterba (1991) [36]

⁶Huberman (2001) [47], Coval and Moskowitz (1999) [26]

tensive review of proposed explanations. Empirical studies⁷ find that home bias is caused by both institutional and behavioral factors.

A more recent research direction explains home bias by means of ambiguity aversion. According to the standard expected utility theory, agents are assumed to make decisions under uncertainty as if they have a prior belief about probability distribution over the set of possible states of the world and then maximize the expected utility according to this distribution. However, individuals often fail to accurately assess such probabilities. Knight [58] suggests that there is an important difference between events with objectively (or subjectively) known probabilities, and events where probabilities are unknown. Uncertainty of the first kind is called *risk*, and uncertainty of the second kind is called *ambiguity* or *Knightian uncertainty*. Ellsberg [32] demonstrates the significance of this distinction by showing that individuals may prefer gambles with specified probabilities over gambles with unknown odds. In the experiment, two urns are given: one contains 50 red balls and 50 black balls, and the other contains 100 red and black balls in unknown proportion. One ball is drawn at random from each urn. In gamble A, the payoff is \$100 if a red ball is drawn and \$0 if a black ball is drawn. In gamble B, the payoff is \$100 if a black ball is drawn and \$0 if a red ball is drawn. When surveyed, many people choose to draw from first urn in both gambles. Such behavior contradicts the standard expected utility paradigm according to which participants form subjective beliefs in the form of a single probability distribution over the composition of balls in the second urn. This experiment has motivated various generalizations of subjective expected utility theory that incorporate ambiguity. One of the most popular approaches is the maxmin multiple prior model of Gilboa and Schmeidler where agents make decisions based on the worst among the many

⁷Bailey, Kumar, and Ng (2005) [10], Karlsson and Norden (2004) [51], Kyrychenko, Shum (2006) [61].

possible probability distributions for any given choice.

This paper develops a two-country model that illustrates how ambiguity about asset payoffs affects asset prices and portfolio holdings. Agents live in a Lucas pure-exchange economy with a safe asset and two country-specific risky assets. There is ambiguity about assets' payoffs, i.e., the agents are uncertain about the exact probability distribution. Similarly to the model developed by Easley and O'Hara [31], ambiguity averse investors act as if they have a set of distributions on payoffs, and select a portfolio in order to maximize their utility over this set of distributions. Agents preferences are characterized by the smooth model of decision making under ambiguity that has been axiomatized by Klibanoff, Marinacci, and Mukerji [57]. The advantage of using the smooth model is that it allows for intermediate values of ambiguity aversion coefficients rather than the extreme cases of minimal expected utility and standard expected utility maximizing agents. Moreover, it also simplifies the analysis due to the smoothness conditions, which makes the model analytically tractable.

All investors possess the same information about the set of possible states and the corresponding returns distribution in each state, but have different beliefs about the likelihood of these states. Optimism and overconfidence refer to the distorted beliefs about expected mean and dispersion of the asset returns distributions, respectively. I show that the difference in beliefs about perceived uncertainty leads to the bias in portfolio holdings. The equilibrium portfolio allocation depends on the degree of ambiguity aversion as well as parameters that characterize uncertainty.

To see whether the equity home bias observed in data can be explained by a less extreme degree of ambiguity aversion, I analyze a numerical example using stylized facts about asset returns. I find that when investors are ambiguity averse

then it is possible that even small difference in beliefs about perceived uncertainty may generate a home bias in portfolio holdings that is close to the data.

The two most closely related papers are Epstein and Miao [33] and Uppal and Wang [77]. Epstein and Miao use a recursive multiple-prior model, a multi-period extension of Gilboa and Schmeidler (1989) maxmin model. They consider agents (countries) who are equally ambiguity averse but have different sets of multiple priors, and hence do not agree on which states are ambiguous. Uppal and Wang [77] study the portfolio choice when an investor accounts for model misspecification. They follow the robust control approach introduced by Hansen, Sargent and Tallarini [42] and Anderson, Hansen, and Sargent [9] where agents use a reference model to differentiate among the priors and maximize the minimum expected utility (minimize the worst case loss) over the set of possible models. Hansen, Sargent, Turmuhambetova, and Williams (2006) established that the model set of robust control can be viewed of as a particular specification of Gilboa and Schmeidler's set of priors. In their paper, Uppal and Wang show that if the confidence about joint stock distribution is low then small differences in the degree of confidence for the marginal payoff distribution will result in a significant underdiversification relative to the standard mean-variance portfolio.

However, the notion of maxmin ambiguity aversion can be viewed as overly pessimistic and may not accurately reflect actual beliefs and preferences. In particular, Bossaerts, Guarnaschelli, Ghirardato and Zame [15] have shown that the attitude toward ambiguity varies across individuals. This suggests that modeling investors' decisions by the maxmin rule may significantly overestimate the effects of ambiguity on asset holding and asset prices. Moreover, Condie [23] shows that in an economy where some agents are ambiguity averse (in the maxmin sense),

and some are standard expected utility maximizers (in the Bayesian sense), the former are unlikely to survive if there is an aggregate risk. This suggests that agents who exhibit extreme ambiguity aversion may decide not to participate in the market, i.e. not to hold any foreign asset at all. Easley and O'Hara [31] study the non-participation of ambiguity averse individuals and examine its implications for the regulation of financial markets.

In contrast to the robust control approach, the preference representation by Klibanoff, Marinacci, and Mukerji has an axiomatic foundation and stays within the state-independent utility framework. Their model allows to smoothly aggregate the decision maker's information about the subjective relevance of each possible probability measure as the true probability measure. This makes it similar to the Bayesian approach. Unlike in Uppal and Wang, in my model the degree of ambiguity aversion is the same for all assets, but investors perceive uncertainty differently for home and foreign assets. Also, my model examines the effect of the ambiguity on the asset prices and derives the upper bound on the degree of ambiguity aversion for participation in financial markets.

The idea that investors have different beliefs about uncertainty is supported by surveys and empirical studies. Several papers in the home bias literature have identified a systematic bias in investors' payoff expectations. French and Poterba (1991) show that observed portfolio holdings could be explained by domestic investors having more optimistic expectations about domestic stocks than about foreign stocks. This has been confirmed by empirical studies for Japan (Shiller, Kon-Ya, and Tsutsui [71] and China (Chen, Kim, Nofsinger, Rui [19]), experimental studies for Germany (Kilka and Weber [54]), and surveys of fund-managers (Strong, Xin [72]). Graham, Harvey and Huang [41] also study the link between

competence and investor behavior where investor competence is measured through survey responses. They argue that the competence effect contributes to home bias. Tourani-Rad and Kirkby [75] investigate investor overconfidence, socialization and the familiarity effect, using a sample of New Zealand investors. They find support for the investor overconfidence theory, using characteristics such as past success, optimism, confidence in one's abilities, investment experience and investment-related knowledge. Lutje and Menkhoff [64] find that belief in an informational advantage and relative payoff optimism towards home assets are the driving forces of home bias. They argue that informational advantage often appears to be a perceived advantage, as fund managers with a home preference do not forecast stock indices better than others, and they rely less on fundamental analysis. Christoffersen and Sarkissian [21] relate geographic location and investor behavior by comparing the performance of U.S. equity mutual funds located in and outside of financial centers. They argue that fund managers in financial centers tend to be more overconfident because of their proximity to private information. Furthermore, Van Nieuwerburgh and Veldkamp [78] argue that even when home investors can learn what foreigners know, they choose not to. They show that learning amplifies information asymmetry since investors profit more from knowing information others do not know.

The remainder of the paper is organized as follows: the model environment is described next in section 2. In section 3, I consider three cases of distorted beliefs: in case 1, there is no uncertainty about the home asset, in case 2, investors are overconfident about the home asset, and in case 3, investors are more optimistic about the home asset. Section 4 provides the equilibrium characterization. Section 5 describes the equilibrium properties. Section 6 presents the numerical results, and section 7 concludes. All proofs are delegated to the Appendix.

3.2 Model

I consider a model with two countries: A and B . One can think about country A as one particular country and about country B as the rest of the world. The total number of agents in both countries is I , λI living in country A and $(1 - \lambda)I$ living in country B , where λ is between zero and one. Agents live in a Lucas pure-exchange economy. There are three assets in the economy: one safe asset m (bond) and two risky assets which are country-specific. The safe asset is the same in both countries and its price and payoff are normalized to one. In addition, each country has a risky asset, which yields a stochastic dividend in every period. The holdings of risky asset in country $k \in \{a, b\}$ ⁸ by an investor i is denoted by x_{ki} . All agents are identical within the country they live in, and each agent from country k is endowed with \bar{m} units of money and \bar{x}_k holdings of the home asset. Hence, the total endowment in the economy is $(I\bar{m}, \lambda I\bar{x}_a, (1 - \lambda)I\bar{x}_b)$. All assets are traded on the international market so investors from both country have access to the market. The price of risky asset of country k is p_k and the payoff is r_k .

There are infinitely many possible (hidden)⁹ states; in each state s , asset payoff r_k is normally distributed with mean $\bar{r}_k(s)$ and variance $\sigma_k^2(s)$. I assume that there is uncertainty only about the mean $\bar{r}_k(s)$ of asset' payoffs, the asset payoff variance is the same in all states: $\sigma_k(s) = \sigma_k$. Furthermore, the payoffs on both assets are independent. Investors do not know which state will be realized, so they form beliefs about a set of possible realizations of mean payoffs for each asset. The investors' beliefs are modeled as second-order priors over the first order probability distributions of asset payoffs. The set of priors represent the ambiguity about asset payoffs. This partition into first and second order distributions captures the

⁸For typographical convenience subscripts \cdot_a and \cdot_b refer to country A and B , respectively.

⁹Agents only observe the realization of assets' returns, the realization of states is not observed.

separation between (objective) information and (subjective) beliefs. See Figure 3.1 for illustration.

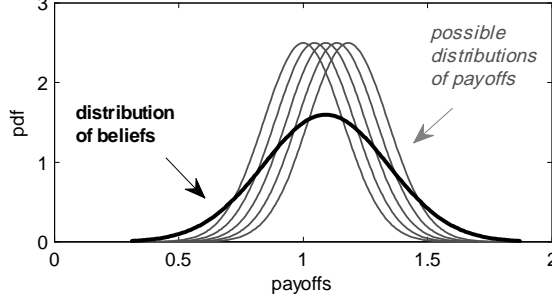


Figure 3.1: Possible distributions of payoffs and distribution of beliefs.

The wealth of each investor i from country k is equal to $w_i = (r_a - p_a)x_{ai} + (r_b - p_b)x_{bi} + \bar{m} + p_i\bar{x}_k$. Investors choose their optimal portfolio (x_{ai}, x_{bi}) to maximize their utility function.

The utility function is adapted from the smooth model of decision making under ambiguity by Klibanoff, Marinacci, and Mukerji (2005). The individual preferences are represented by

$$U(w) = E_{\mu} [\phi(E_{\pi_s}[u(w)|s_n])] \quad (3.1)$$

where $u(\cdot)$ is a von Neumann-Morgenstern utility function, π_s is a known probability distribution in each state s , and μ is subjective probability distribution over the possible probabilities π_s . The subjective prior μ weights the importance of each distribution π_s reflecting an investor's beliefs about the likelihood of each state. The increasing function ϕ characterizes the attitudes towards ambiguity. The degree of ambiguity aversion is defined as $\alpha(y) = -\phi''(y)/\phi'(y)$. If function ϕ is concave then it characterizes ambiguity aversion, which is defined as an aversion to mean preserving spreads in μ . If function ϕ is linear then the reduction of compound lotteries can be applied and it becomes equivalent to the standard subjective

expected utility. The model of maxmin expected utility: $U(\cdot) = \min_{\pi_s} E_{\pi_s}[u(\cdot)]$ may be seen as an extreme case of my model with infinite degree of ambiguity aversion.

The smooth model allows the separation between ambiguity (a decision maker's subjective beliefs μ) and ambiguity attitude (a characteristic of the decision maker's preferences α). It smoothly aggregates the decision maker's information about likelihood of each possible probability distribution, consequently, the indifference curves are smooth rather than kinked. Note that in maxmin models, the decision maker only looks at the the worst value.

I assume $u(w) = -e^{-\gamma w}$ is a CARA utility function where γ is the degree of risk aversion. If investors are ambiguity neutral then ϕ is linear: $\phi(y) = y$; if investors are ambiguity averse then $\phi(y) = -e^{-\alpha y}$ where α is the degree of ambiguity aversion. These assumptions on investors preferences together with the normally distributed payoffs allow to derive results for prices and asset holdings in closed-form.

3.3 Overconfidence and Optimism

Investors believe that possible mean payoffs $(\bar{r}_a(s), \bar{r}_b(s))$ are jointly normally distributed with mean (\bar{r}_a, \bar{r}_b) and covariance matrix $\begin{bmatrix} \delta_a^2 & \delta_{ab} \\ \delta_{ab} & \delta_b^2 \end{bmatrix}$, where δ_k^2 characterizes the dispersion of possible distributions for each asset and δ_{ab} characterizes the correlation between states. This correlation reflects the investors' beliefs that if more favorable state is realized for one country than it is more likely to be realized for the other one as well. The correlation is based on investors' expectations

rather than fundamentals, allowing to capture a possible contagion effect between the two countries. The investors' expectations (beliefs) are the driving force of at least some episodes of financial market contagion. If investors expect the asset returns in different countries to be correlated then their investment decisions create links between otherwise separate markets that may lead to financial contagion. (Goldstein and Pauzner [38] and Keister[53])

The beliefs about the dispersion of payoffs distributions depend on whether the asset is domestic or foreign. Investors believe that there is less uncertainty about the home asset than about the foreign asset, and they are more optimistic about the payoffs on the home asset. These assumptions are supported by the findings of Kilka and Weber [54]. They conduct a cross-country study in Germany and the U.S. to investigate whether people's subjective probability distributions on average exhibit systematic differences in location and in dispersion. Their results show that people consider themselves to be on average more competent in forecasting domestic stock prices than in forecasting foreign stocks prices. Subjective probability distributions of stock payoffs are significantly less dispersed for domestic stocks (associated with high confidence levels) than for foreign stocks (associated with low confidence levels). Furthermore, domestic stocks are judged significantly more optimistically than foreign stocks. These observed patterns are consistent with biases in individual judgment documented by psychological research (Heath and Tversky [44]).

In this paper, I refer to *optimism* as distorted beliefs about the expected mean and *overconfidence* as distorted beliefs about the variance of mean returns distribution. The optimistic investors believe the expected mean is larger than the true value, in particular, beliefs about the home asset payoffs first order stochasti-

cally dominates beliefs about the foreign asset payoffs. The overconfident investors overestimate the precision of probability distribution of asset returns, in particular, beliefs about the home asset payoffs second order stochastically dominates beliefs about the foreign asset payoffs.

First, I will consider the economy with an extreme version of overconfidence in which investors completely ignore uncertainty about the home asset, and consequently they behave as standard expected utility maximizers with respect to home asset. In the next case, investors believe there is less uncertainty about the home asset, i.e. the dispersion of possible distributions is smaller for the home asset than for the foreign asset. Third, I consider the model where investors face the same uncertainty about home and foreign assets but they are more optimistic about payoffs on the home asset.

3.3.1 Case 1. No Uncertainty about Home Asset

First, consider the extreme case when investors completely ignore uncertainty about the home asset but not about the foreign asset. Investors form a single prior about payoffs on the home asset, i.e., instead of considering all possible distributions they put a mass point weight on one average distribution with mean \bar{r}_k and variance σ_k^2 . In this case investors can exhibit any degree of ambiguity aversion α with respect to the home asset but it is irrelevant since their beliefs about asset payoffs consist of a single prior. Consequently, results are equivalent to having a linear ϕ -function with respect to the asset payoffs, and therefore, it is equivalent to ambiguity neutrality with respect to that asset. For foreign asset, investors take into consideration all possible distributions and, therefore, the degree of ambiguity aversion α matters. Effectively, investors are ambiguity neutral

with respect to domestic asset and ambiguity averse with respect to the foreign asset. See Figure 3.2 for illustration. The asset payoffs are normally distributed with some mean $\bar{r}_k(s)$ and variance $\sigma_k^2 : r_k \sim N(\bar{r}_k(s), \sigma_k^2)$, $k = a, b$. Investors believe that the possible mean payoffs are equal to the average of mean payoffs if it is a home asset, or normally distributed with mean \bar{r}_k and variance δ_k^2 , if it is a foreign asset,

$$\bar{r}_k(s) \begin{cases} = \bar{r}_k & \text{if } k \text{ is home asset} \\ \sim N(\bar{r}_k, \delta_k^2) & \text{if } k \text{ is foreign asset} \end{cases}. \quad (3.2)$$

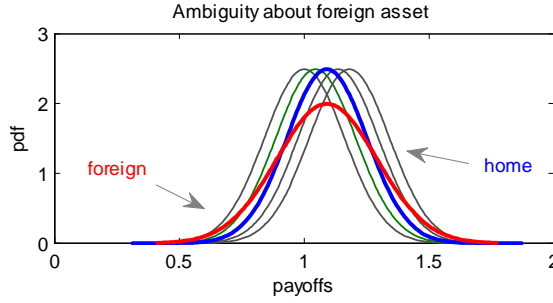


Figure 3.2: Ambiguity about foreign asset.

In the competitive equilibrium, investors choose portfolio holdings to maximize their expected utility, and prices are determined such that markets clear. Since asset payoffs are assumed to be distributed normally and investors have a CARA utility function, maximization problem can be expressed in terms of mean and variance.

An investor i from country k solve the following optimization problem:

$$\max_{x_{hi}, x_{fi}} \left\{ (\bar{r}_h - p_h)x_{hi} + (\bar{r}_f - p_f)x_{fi} - \frac{1}{2}\gamma (\sigma_h^2 x_{hi}^2 + \sigma_f^2 x_{fi}^2 + \alpha \delta_f^2 x_{fi}^2) \right\} \quad (3.3)$$

where $h, f \in \{a, b\}$ denote respectively the home country and the foreign country for investor i .

From now on, denote investors from country B by j and investors from country A by i . Then the optimal demands for the home and the foreign assets are given by

$$\begin{aligned} \text{country } A \text{ investors } i: \quad x_{ai} &= \frac{\bar{r}_a - p_a}{\gamma \sigma_a^2}; & x_{bi} &= \frac{\bar{r}_b - p_b}{\gamma(\sigma_b^2 + \alpha \delta_b^2)}; \\ \text{country } B \text{ investors } j: \quad x_{aj} &= \frac{\bar{r}_a - p_a}{\gamma(\sigma_a^2 + \alpha \delta_a^2)}; & x_{bj} &= \frac{\bar{r}_b - p_b}{\gamma \sigma_b^2}. \end{aligned} \quad (3.4)$$

Note that the difference in the demand functions for home and foreign assets depends on $\alpha \delta_k^2$ where α represents the ambiguity attitude and δ_k^2 - the difference in beliefs about uncertainty of asset payoffs. If investors consider asset payoffs to be more uncertain then they demand less of that asset.

In equilibrium, the demand for optimal asset holdings should satisfy the market clearing conditions: the aggregate demand for optimal asset holdings should be equal to the total endowment,

$$\begin{aligned} \lambda x_{ai} + (1 - \lambda) x_{aj} &= \lambda \bar{x}_a; \\ \lambda x_{bi} + (1 - \lambda) x_{bj} &= (1 - \lambda) \bar{x}_b. \end{aligned} \quad (3.5)$$

For investor i from country A, the equilibrium portfolio holdings (x_{ai}, x_{bi}) of asset a and b are given by

$$\begin{aligned} x_{ai} &= \lambda \bar{x}_a \frac{\sigma_a^2 + \alpha \delta_a^2}{\sigma_a^2 + \alpha \lambda \delta_a^2}; \\ x_{bi} &= (1 - \lambda) \bar{x}_b \frac{\sigma_b^2}{\sigma_b^2 + (1 - \lambda) \alpha \delta_b^2}. \end{aligned} \quad (3.6)$$

For investor j from country B, the equilibrium portfolio holdings (x_{aj}, x_{bj}) of asset a and b are given by

$$\begin{aligned} x_{aj} &= \lambda \bar{x}_a \frac{\sigma_a^2}{\sigma_a^2 + \lambda \alpha \delta_a^2}; \\ x_{bj} &= (1 - \lambda) \bar{x}_b \frac{\sigma_b^2 + \alpha \delta_b^2}{\sigma_b^2 + \alpha (1 - \lambda) \delta_b^2}. \end{aligned} \quad (3.7)$$

If there is no ambiguity then the holding of each country's asset is the same for investors from both countries, and should be equal to the per capita supply of that asset, i.e., $x_{ki} = x_{kj} = \lambda_k \bar{x}_k$ where $\lambda_a = \lambda$ and $\lambda_b = 1 - \lambda$. However, if there is a difference in perceived uncertainty about the home and the foreign asset then portfolio holdings are biased towards the home asset: the holdings of home asset x_h is larger than its market capitalization $\lambda_h \bar{x}_h$ and the holdings of foreign asset x_f is smaller than its market capitalization $\lambda_f \bar{x}_f$ where $h, f \in \{a, b\}$.

Note it is not the ambiguity by itself that causes the bias in portfolio holdings but the distortion in beliefs. If investors perceive both assets as equally ambiguous then their asset holdings are proportional to the market capitalization. Moreover, Gollier [40] identified the conditions when the increase in ambiguity aversion can lead to the increase in demand for the ambiguous risky asset.

3.3.2 Case 2. Overconfidence about Home Asset

In this section I relax the assumption that investors completely ignore uncertainty about the home asset, i.e., they behave as if they know the true distribution. Suppose investors are now effectively ambiguity averse with respect to both assets, home and foreign, but they believe there is less uncertainty about the home asset. See Figure 3.3 for illustration. In their beliefs they put more weight on distributions that are close to the mean distribution, i.e., the dispersion of possible distributions is smaller for the home asset than for the foreign asset. The payoffs of asset from country k are normally distributed with some mean $\bar{r}_k(s)$ and variance σ_k^2 : $r_k \sim N(\bar{r}_k(s), \sigma_k^2)$, $k = a, b$. Investors believe that possible mean payoffs are normally distributed with mean \bar{r}_k and perceived variance δ_{kh}^2 if k is a home asset

and perceived variance δ_{kf}^2 if k is a home asset, where $\delta_{kh} < \delta_{kf}$.¹⁰

$$\bar{r}_k(s) \sim \begin{cases} N(\bar{r}_k, \delta_{kh}^2) & \text{if } k \text{ is a home asset} \\ N(\bar{r}_k, \delta_{kf}^2) & \text{if } k \text{ is a foreign asset} \end{cases}$$

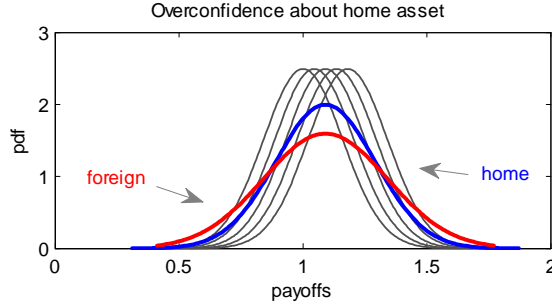


Figure 3.3: Overconfidence about home asset.

If the interstate correlation between assets is zero ($\delta_{ab} = 0$)¹¹, then the equilibrium prices are given by

$$\begin{aligned} p_a &= \bar{r}_a - \lambda \bar{x}_a \gamma \frac{(\sigma_a^2 + \alpha \delta_{ah}^2)(\sigma_a^2 + \alpha \delta_{af}^2)}{\sigma_a^2 + \alpha (\lambda \delta_{af}^2 + (1 - \lambda) \delta_{ah}^2)}; \\ p_b &= \bar{r}_b - (1 - \lambda) \bar{x}_b \gamma \frac{(\sigma_b^2 + \alpha \delta_{bh}^2)(\sigma_b^2 + \alpha \delta_{bf}^2)}{\sigma_b^2 + \alpha (\lambda \delta_{bh}^2 + (1 - \lambda) \delta_{bf}^2)}. \end{aligned} \quad (3.8)$$

For investor i from country A, the equilibrium portfolio holdings (x_{ai}, x_{bi}) of asset a and b are given by

$$\begin{aligned} x_{ai} &= \lambda \bar{x}_a \frac{\sigma_a^2 + \alpha \delta_{af}^2}{\sigma_a^2 + \alpha (\lambda \delta_{af}^2 + (1 - \lambda) \delta_{ah}^2)}; \\ x_{bi} &= (1 - \lambda) \bar{x}_b \frac{\sigma_b^2 + \alpha \delta_{bh}^2}{\sigma_b^2 + \alpha (\lambda \delta_{bh}^2 + (1 - \lambda) \delta_{bf}^2)}. \end{aligned} \quad (3.9)$$

For investor j from country B, the equilibrium portfolio holdings (x_{aj}, x_{bj}) of

¹⁰That is, δ_{ah}^2 denotes the perceived variance of asset a by investors from country A and δ_{af}^2 denotes the perceived variance of asset a by investors from country B. Similarly for the perceived variance of the asset b .

¹¹See section C.2 of the Appendix for the solution when $\delta_{ab} \neq 0$.

asset a and b are given by

$$\begin{aligned} x_{aj} &= \lambda \bar{x}_a \frac{\sigma_a^2 + \alpha \delta_{ah}^2}{\sigma_a^2 + \alpha (\lambda \delta_{af}^2 + (1 - \lambda) \delta_{ah}^2)}; \\ x_{bj} &= (1 - \lambda) \bar{x}_b \frac{\sigma_b^2 + \alpha \delta_{bf}^2}{\sigma_b^2 + \alpha (\lambda \delta_{bh}^2 + (1 - \lambda) \delta_{bf}^2)}. \end{aligned} \quad (3.10)$$

As in the previous case, the portfolio holdings are biased towards the home asset. For the home asset, equilibrium holdings are larger than the market capitalization: $x_{ai} > \lambda \bar{x}_a$ and $x_{bj} > (1 - \lambda) \bar{x}_b$. For the foreign asset, equilibrium holdings are smaller than the market capitalization: $x_{bi} < \lambda \bar{x}_b$ and $x_{aj} < (1 - \lambda) \bar{x}_a$. When investors face uncertainty (ambiguity) about both, home and foreign, assets, their portfolio holdings will be biased towards the home asset if they are overconfident about the home asset relative to the foreign. The extent of the bias depends on the difference in perceived uncertainty about two assets: $\delta_{kf}^2 > \delta_{kh}^2$.

3.3.3 Case 3. Optimism about Home Asset

Now suppose investors face the same uncertainty about the home and the foreign asset but they are more optimistic about payoffs on the home asset. Investors have distorted beliefs about the second-order distributions over states, with respect to the home asset they are optimistic about the realization of states with realization of payoffs mean above average, and with respect to the foreign asset, investors think that the states with realization of payoffs mean which is below average is more likely. See Figure 3.3 for illustration.

The asset payoffs are normally distributed with some mean $\bar{r}_k(s)$ and variance σ_k^2 : $r_k \sim N(\bar{r}_k(s), \sigma_k^2)$, $k = a, b$. Investors believe that possible mean payoffs are normally distributed with mean \bar{r}_k and variance $\delta_{k\cdot}^2$, i.e., $\bar{r}_k(s) \sim$

$$\begin{cases} N(\bar{r}_{kh}, \delta_k^2) & \text{if } k \text{ is a home asset} \\ N(\bar{r}_{kf}, \delta_k^2) & \text{if } k \text{ is a foreign asset} \end{cases} \quad \text{where } \bar{r}_{kh} > \bar{r}_{kf}.$$

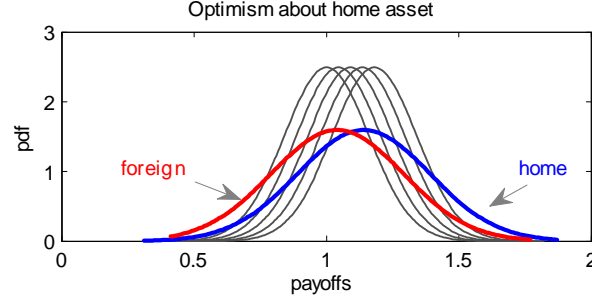


Figure 3.4: Optimism about home asset.

In this case, the equilibrium prices are given by

$$p_a = (\lambda \bar{r}_{ah} + (1 - \lambda) \bar{r}_{af}) - \lambda \bar{x}_a \gamma (\sigma_a^2 + \alpha \delta_a^2) - (1 - \lambda) \bar{x}_b \gamma \alpha \delta_{ab}; \quad (3.11)$$

$$p_b = (\lambda \bar{r}_{bf} + (1 - \lambda) \bar{r}_{bh}) - (1 - \lambda) \bar{x}_b \gamma (\sigma_b^2 + \alpha \delta_b^2) - \lambda \bar{x}_a \gamma \alpha \delta_{ab}.$$

For investor i from country A, the equilibrium portfolio holdings (x_{ai}, x_{bi}) of asset a and b are given by

$$x_{ai} = \lambda \bar{x}_a \left(1 + \frac{(1 - \lambda)(\bar{r}_{ah} - \bar{r}_{af})(\sigma_b^2 + \alpha \delta_b^2) + (1 - \lambda)(\bar{r}_{bh} - \bar{r}_{bf})\alpha \delta_{ab}}{\gamma((\sigma_a^2 + \alpha \delta_a^2)(\sigma_b^2 + \alpha \delta_b^2) - (\alpha \delta_{ab})^2)} \right); \quad (3.12)$$

$$x_{bi} = (1 - \lambda) \bar{x}_b \left(1 - \frac{(1 - \lambda)(\bar{r}_{bh} - \bar{r}_{bf})(\sigma_a^2 + \alpha \delta_a^2) + (1 - \lambda)(\bar{r}_{ah} - \bar{r}_{af})\alpha \delta_{ab}}{\gamma((\sigma_a^2 + \alpha \delta_a^2)(\sigma_b^2 + \alpha \delta_b^2) - (\alpha \delta_{ab})^2)} \right).$$

For investor j from country B, the equilibrium portfolio holdings (x_{aj}, x_{bj}) of asset a and b are given by

$$x_{aj} = \lambda \bar{x}_a \left(1 - \frac{\lambda(\bar{r}_{ah} - \bar{r}_{af})(\sigma_b^2 + \alpha \delta_b^2) + \lambda(\bar{r}_{bh} - \bar{r}_{bf})\alpha \delta_{ab}}{\gamma((\sigma_a^2 + \alpha \delta_a^2)(\sigma_b^2 + \alpha \delta_b^2) - (\alpha \delta_{ab})^2)} \right); \quad (3.13)$$

$$x_{bj} = (1 - \lambda) \bar{x}_b \left(1 + \frac{\lambda(\bar{r}_{bh} - \bar{r}_{bf})(\sigma_a^2 + \alpha \delta_a^2) + \lambda(\bar{r}_{ah} - \bar{r}_{af})\alpha \delta_{ab}}{\gamma((\sigma_a^2 + \alpha \delta_a^2)(\sigma_b^2 + \alpha \delta_b^2) - (\alpha \delta_{ab})^2)} \right).$$

As expected, there is a bias towards the home asset in portfolio holdings. The holdings of home asset is larger than its market capitalization ($x_h > \lambda_h \bar{x}_h$) and the holdings of foreign asset is smaller than its market capitalization ($x_f < \lambda_f \bar{x}_f$).

3.4 Portfolio Holdings and Ambiguity

The Proposition below summarizes and generalizes results considered in the previously analyzed cases.

Proposition 5 *If investors are ambiguity averse with respect to both assets and they are overconfident about the home asset relative to the foreign asset: $\delta_{kh} < \delta_{kf}$, $k = a, b$, or they are optimistic about the home asset relative to the foreign asset: $\bar{r}_{kh} > \bar{r}_{kf}$, $k = a, b$ then investors will choose their portfolio so that the proportion of the home asset is larger than its market share and the proportion of the foreign asset is smaller than its market share:*

$$\begin{aligned} \frac{x_{ai}}{x_{ai} + x_{bi}} &> \frac{\lambda \bar{x}_a}{\lambda \bar{x}_a + (1 - \lambda) \bar{x}_b}; \\ \frac{x_{aj}}{x_{aj} + x_{bj}} &< \frac{\lambda \bar{x}_a}{\lambda \bar{x}_a + (1 - \lambda) \bar{x}_b}. \end{aligned}$$

In equilibrium, the price for the home asset is higher than the expected price for the home asset, and the price for the foreign asset is lower than the expected price for the foreign asset based on the investor's perceived ambiguity.

For simplicity, consider again the case 1 when investors completely ignore the uncertainty about the home asset. The investors from country A believe that the price for their home asset should be $p_{ah} = \bar{r}_a - \lambda \bar{x}_a \gamma \sigma_a^2$ which is the equilibrium price

when all investors ignore uncertainty about the asset.¹² However, since foreign investors view the asset a as ambiguous the equilibrium price is given by

$$p_a = \bar{r}_a - \lambda \bar{x}_a \gamma \sigma_a^2 \frac{\sigma_a^2 + \alpha \delta_a^2}{\sigma_a^2 + \alpha \lambda \delta_a^2} \quad (3.14)$$

The equilibrium price is higher than the price expected by home investors. Therefore, country A believe that asset a is overpriced, so they have incentive to hold more of the home asset.

Similarly, country B investors believe that the price for asset a should be $p_{af} = \bar{r}_a - \lambda \bar{x}_a \gamma (\sigma_a + \alpha \delta_a^2)$, which is the equilibrium price when all investors view the asset a as ambiguous. This expected price p_{af} is lower than the equilibrium price p_a . Since country B investors believe that foreign asset is underpriced, they hold less of it in their portfolio.

Therefore, investors believe that asset a is overpriced if it is home asset, and underpriced if it is a foreign asset. The same conclusions hold for asset b due to the symmetry. Hence, investors see the arbitrage opportunities and as a result hold more of the home asset and less of the foreign asset relative to their respective market capitalization weights. Therefore, the equity home bias arises as consequences of investors difference in beliefs about uncertainty of the asset returns. The same conclusions apply for both asset in more general cases when investors are optimistic and overconfident about the home asset relative the foreign asset.

Unlike in models with asymmetric information, in this framework prices are not informative. If prices are informative than informational advantage should eventually be arbitrated away through the active trading. In this model, the difference in actual vs expected asset price reflect the investors' difference in beliefs.

¹²It is equivalent to the case where all investors behave as standard expected utility maximizers.

When an investor thinks that others have wrong beliefs then he has no incentive to adjust his portfolio allocation after observing prices different from what is expected.

3.5 Equilibrium Properties

3.5.1 Comparative Statics

Next proposition summarizes the effects of change in the degree of ambiguity aversion, difference in the perceived mean returns and perceived dispersions of mean asset returns, correlation of asset returns, and market capitalization.

Proposition 6 *The equity home bias is larger if (i) market capitalization λ is smaller; (ii) degree of ambiguity aversion α is higher; (iii) difference in the perceived mean returns $\Delta \bar{r}_k \equiv \bar{r}_{kh} - \bar{r}_{kf}$; (iv) difference in the perceived dispersions of mean asset returns $\Delta \delta_k \equiv \delta_{kf} - \delta_{kh}$ are larger; and (v) correlation of asset returns $\rho_{ab} \equiv \frac{\delta_{ab}}{\delta_a \delta_b}$ is positive.*

The first result explains why countries with small market capitalization (like Canada or Scandinavian countries) exhibit significantly larger home bias relative to their market capitalization share. If investors from one country dominate the market then they have a large impact on equilibrium asset prices. So the deviation between equilibrium prices and the expected prices is smaller, therefore, the portfolio holdings are closer to the market capitalization weights. Similarly, if the proportion of investors from one country is relatively small than their asset holding will be strongly biased towards the home asset.

Table 3.1: Comparative statics summary.

\backslash	$\alpha \uparrow$	$\Delta\delta_a \uparrow$	$\Delta\bar{r}_a \uparrow$	$\rho_{ab} \uparrow$	$\lambda \uparrow$
p_a	\downarrow	\uparrow	\uparrow	\downarrow	\downarrow
x_{ah}	$?$	\uparrow	\uparrow	\downarrow	\downarrow
x_{af}	\downarrow	\downarrow	\downarrow	\uparrow	\uparrow

The next three results are intuitive: these parameters $(\alpha, \Delta\delta_k, \Delta\bar{r}_k)$ directly contribute to the difference in perceived uncertainty about two assets. If any of these parameters increase it will lead to the increase of the home bias. If the degree of ambiguity aversion increases then the prices of both assets go down, the holding of the home asset may increase or decrease and the foreign asset holding decreases. Overall, the equity home bias becomes larger. If for a given asset the difference in the perceived dispersions increases then its equilibrium price goes down. The holding of this asset decreases if it is a foreign asset, and increases if it is a home asset. Therefore, the equity home bias becomes larger. If for a given asset the difference in the perceived mean returns increases then its equilibrium price goes up. The holding of this asset decreases if it is a foreign asset, and increases if it is a home asset. Therefore, the equity home bias becomes larger.

If investors believe that the state realization of asset return distributions are correlated then they will have incentive to hedge. If the correlation is negative then investors will diversify more due to hedging motives, hence, the equity home bias is smaller. On the other hand, the positive correlation reduces benefits from the diversification and leads to the larger home bias.

Table 3.1 summarizes all effects of possible changes in parameters of asset a on the price and asset holdings.

3.5.2 Non-participation

Another implication of my model is that there is an upper bound on the degree of ambiguity aversion that comes from the requirement of the asset price to be non-negative. Investors will choose to participate in the market only if they believe that the price for the foreign asset is positive. This means that if investors have a degree of ambiguity aversion α such that

$$\alpha \geq \frac{\bar{r}_f - \lambda_f \bar{x}_f \gamma \sigma_f^2}{\lambda_f \bar{x}_f \gamma \delta_{kf}^2}, \quad (3.15)$$

they will choose not hold any of the foreign asset. If the degree of ambiguity aversion is too large, investor may prefer to hold on to their endowment of home asset, rather than bear ambiguity associated with the foreign asset. This upper bound out the participation of agents with maxmin type of preferences if there are no restrictions on the set of possible means for asset returns. The upper bound on the degree of ambiguity aversion is inversely related to the perceived ambiguity about the foreign asset characterized by δ_{kf}^2 . Therefore, reducing ambiguity about the foreign asset will increase its portfolio share and, hence, decrease the home bias. Easley and O'Hara [30] and [31] demonstrate the potential benefits from reducing ambiguity, and examine the implications of the presence of ambiguity averse traders for market regulations such as deposit insurance and securities regulation.

3.5.3 Equity Premium

Define the equity premium as $EP \equiv E[r_k]/p - 1$. If all investors are ambiguity neutral (SEU) then equity premium is

$$EP_{SEU} = \frac{\bar{r}_k}{\bar{r}_k - \lambda_k \bar{x}_k \gamma \sigma_k^2} - 1. \quad (3.16)$$

If all investors are ambiguity averse (AA) then equity premium becomes

$$EP_{AA} = \frac{\bar{r}_k}{\bar{r}_k - \lambda_k \bar{x}_k \gamma (\sigma_k^2 + \alpha \delta_k^2)} - 1. \quad (3.17)$$

The equity premium is higher under ambiguity, and as degree of ambiguity aversion or the dispersion of possible distribution increases, the premium becomes larger. In the presence of ambiguity, risk sharing opportunities offered by financial markets become less complete which could lead to a no-trade equilibrium (Mukurji and Tallon [68]). The positive effect of ambiguity on the equity premium has been addressed as an application by several papers on decision theory under uncertainty (Epstein and Wang [34], Chen and Epstein [20]).

3.6 Numerical Results

In this section I will investigate the quantitative joint effect¹³ of optimism and overconfidence on asset holdings. The asset returns are normally distributed with some mean $\bar{r}_k(s)$ and variance $\sigma_k^2 : r_k \sim N(\bar{r}_k(s), \sigma_k^2)$, $k = a, b$. Investors believe that the possible mean returns are normally distributed with mean \bar{r}_k and variance δ_k^2 , i.e., $\bar{r}_k(s) \sim \begin{cases} N(\bar{r}_{kh}, \delta_{kh}^2) & \text{if } k \text{ is a home asset} \\ N(\bar{r}_{kf}, \delta_{kf}^2) & \text{if } k \text{ is a foreign asset} \end{cases}$ where $\delta_{kh} < \delta_{kf}$ and $\bar{r}_{kh} > \bar{r}_{kf}$.

¹³The theoretical results are presented in the proof of Proposition 1 (Section C.3 of the Appendix).

Table 3.2: Holdings of home asset when market capitalization is 50%.

$\lambda = 0.5$	$\alpha = 1$				$\alpha = 2$			
$\Delta\delta_k \setminus \Delta\bar{r}_k$	0%	1%	2%	5%	0%	1%	2%	5%
0%	50	53.10	56.19	65.48	50	53.10	56.19	65.48
1%	51.30	54.39	57.47	66.45	52.05	55.14	58.20	67.31
2%	52.60	55.67	58.73	67.78	54.10	57.15	60.18	69.07
5%	56.45	59.45	62.41	71.03	60.06	62.96	65.79	73.91
10%	62.56	65.37	68.10	75.83	68.93	71.42	73.81	80.43

$\lambda = 0.5$	$\alpha = 5$				$\alpha = 10$			
$\Delta\delta_k \setminus \Delta\bar{r}_k$	0%	1%	2%	5%	0%	1%	2%	5%
0%	50	53.10	56.19	65.48	50	53.10	56.19	65.48
1%	53.24	56.31	59.35	68.34	54.58	57.62	64.84	69.48
2%	56.45	59.45	62.41	71.03	59.03	61.97	75.43	73.11
5%	65.41	68.09	70.68	77.93	70.77	73.15	75.43	81.70
10%	77.02	78.97	80.81	85.81	83.44	84.87	86.22	89.84

I assume the following stylized facts:¹⁴ expected asset return r_t^m is 9%, asset standard deviation $\sigma(r_t^m)$ is 16%, coefficient of risk aversion is equal to 2. According to Ahearne, Grier, and Warnock (2004), the US market capitalization is about 48.3% ($\lambda \simeq 0.5$) and the estimated home asset holding is about 89.9%.

Table 3.2 presents the home asset holdings for several values of the difference in perceived mean returns $\Delta\bar{r}_k$ and perceived dispersions $\Delta\delta_k$, for different degrees of ambiguity aversion. The perceived mean returns for the home asset is $\bar{r}_{kh} = 1.09 + \Delta\bar{r}_k/2$, and for the foreign asset it is $\bar{r}_{kf} = 1.09 - \Delta\bar{r}_k/2$. The exact values of dispersions are chosen to match the equity premium of 8%; these values are presented in Table 3.3.

As the degree of ambiguity aversion increases, the bias toward the home asset becomes larger for any given difference in perceived mean returns $\Delta\bar{r}_k$ and perceived dispersions $\Delta\delta_k$. The difference in mean returns $\Delta\bar{r}_k$ contributes more to the bias than the difference in perceived dispersions $\Delta\delta_k$. The significant portion

¹⁴Cochrane [22]

Table 3.3: Perceived dispersion of foreign asset when market capitalization is 50%.

$\lambda = 0.5$	$\alpha = 1$				$\alpha = 2$			
$\Delta\delta_k \backslash \Delta\bar{r}_k$	0%	1%	2%	5%	0%	1%	2%	5%
0%	23.48	23.48	23.48	23.48	16.6	16.6	16.6	16.6
1%	23.99	24.02	24.05	24.08	17.12	17.15	17.18	17.27
2%	24.52	24.58	24.64	24.70	17.66	17.72	17.78	17.96
5%	26.21	26.36	26.51	26.66	19.43	19.58	19.72	20.15
10%	29.38	29.67	29.96	30.24	22.84	23.12	23.39	24.17

$\lambda = 0.5$	$\alpha = 5$				$\alpha = 10$			
$\Delta\delta_k \backslash \Delta\bar{r}_k$	0%	1%	2%	5%	0%	1%	2%	5%
0%	10.5	10.5	10.5	10.5	7.43	7.43	7.43	7.43
1%	11.02	11.17	11.08	11.17	7.95	7.99	8.02	8.11
2%	11.58	11.88	11.70	11.88	8.54	8.60	8.66	8.83
5%	13.50	14.19	13.79	14.19	10.61	10.74	10.88	11.25
10%	17.32	18.49	17.82	18.49	14.77	14.99	15.20	15.77

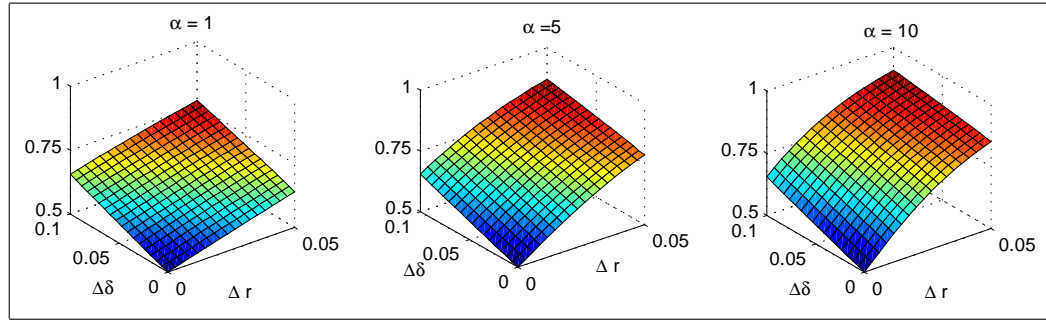


Figure 3.5: Portfolio holdings of home asset for $\alpha = 1, 5, 10$ and $\lambda = 0.5$.

of the bias can be explained with relatively a small degree of ambiguity aversion and differences in beliefs within 5%. It is possible to match exactly the US domestic asset holding observed in data but it requires large (but still finite) degree of ambiguity aversion or large differences in beliefs. The dispersion levels required to match the equity premium is smaller for a higher degree of ambiguity aversion.

Figure 3.5 presents the home asset holdings as a function of the difference in perceived mean returns $\Delta\bar{r}_k$ and dispersion $\Delta\delta_k$ when $\alpha = 1$ (ambiguity neutrality), $\alpha = 5$ and $\alpha = 10$ when $\lambda = 0.5$. The difference in perceived mean returns $\Delta\bar{r}_k$ ranges from 0% to 5% and perceived dispersions $\Delta\delta_k$ ranges from 0% to 10%.

Table 3.4: Holdings of home asset when market capitalization is 10%.

$\lambda = 0.1$	$\alpha = 1$				$\alpha = 2$			
$\Delta\delta_k \setminus \Delta\bar{r}_k$	0%	1%	2%	5%	0%	1%	2%	5%
0%	10	15.81	21.56	38.52	10	15.81	21.56	38.52
1%	10.99	17.02	22.94	40.18	11.42	17.54	23.53	40.88
2%	12.05	18.29	24.38	41.86	12.99	19.39	25.61	43.26
5%	15.68	22.48	28.98	46.92	18.56	25.67	32.36	50.38
10%	23.12	30.52	37.36	55.14	30.34	37.84	44.60	61.44

$\lambda = 0.1$	$\alpha = 5$				$\alpha = 10$			
$\Delta\delta_k \setminus \Delta\bar{r}_k$	0%	1%	2%	5%	0%	1%	2%	5%
0%	10	15.81	21.56	38.52	10	15.81	21.56	38.52
1%	12.31	18.60	24.73	42.26	13.38	19.85	26.11	43.83
2%	14.99	21.70	28.14	46.04	17.49	24.49	31.13	49.14
5%	25.10	32.57	39.41	56.99	33.42	40.86	47.50	63.81
10%	45.77	52.46	58.28	72.00	61.55	66.54	70.80	80.64

As proportion λ of country A investors decreases, the equity home bias becomes larger. The intuition is the following: if investors from one country dominate the market then they have a large impact on equilibrium asset prices. So the deviation between equilibrium prices and the expected prices is smaller, hence, the portfolio holdings are closer to the market capitalization weights. Similarly, if the proportion of investors from one country is relatively small than their asset holdings will be strongly biased towards the home asset. Table ?? presents the home asset holdings when the market capitalization is 10%, the values of dispersions are presented in Table 3.5

Figure 3.6 presents the home asset holdings as a function of the difference in perceived mean returns $\Delta\bar{r}_k$ and dispersion $\Delta\delta_k$ when $\alpha = 1$ (ambiguity neutrality) and $\alpha = 10$ for $\lambda = 0.1$. The difference in perceived mean returns $\Delta\bar{r}_k$ and perceived dispersions $\Delta\delta_k$ ranges from 0% to 5%.

Table 3.5: Perceived dispersion of foreign asset when market capitalization is 10%.

$\lambda = 0.1$	$\alpha = 1$				$\alpha = 2$			
$\Delta\delta_k \setminus \Delta\bar{r}_k$	0%	1%	2%	5%	0%	1%	2%	5%
0%	22.67	22.77	22.85	23.09	16.03	16.10	16.16	16.33
1%	23.51	23.61	23.71	23.97	16.87	16.95	17.02	17.21
2%	24.35	24.46	24.57	24.86	17.71	17.80	17.88	18.10
5%	26.90	27.04	27.18	27.55	20.28	20.39	20.50	20.80
10%	31.20	31.39	31.57	32.07	24.61	24.77	24.92	25.35

	$\alpha = 5$				$\alpha = 10$			
$\Delta\delta_k \setminus \Delta\bar{r}_k$	0%	1%	2%	5%	0%	1%	2%	5%
0%	10.14	10.18	10.22	10.32	7.17	7.20	7.23	7.30
1%	10.98	11.03	11.08	11.21	8.01	8.05	8.09	8.19
2%	11.83	11.89	11.95	12.11	8.86	8.91	8.96	9.09
5%	14.50	14.50	14.58	14.82	11.47	11.54	11.62	11.82
10%	18.92	18.92	19.05	19.41	15.89	16.01	16.12	16.45

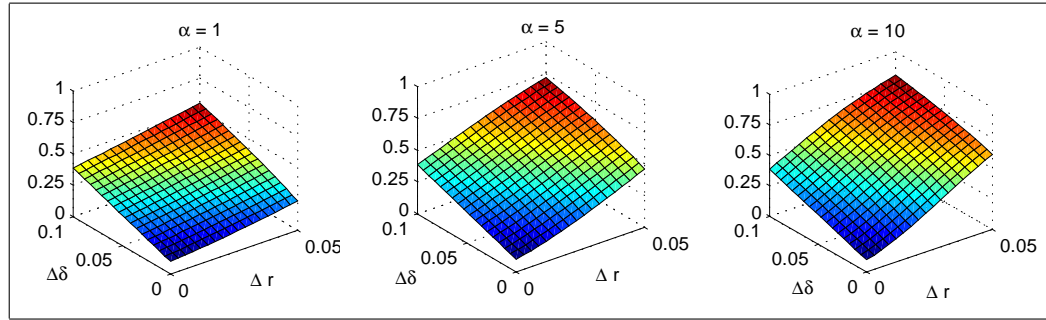


Figure 3.6: Portfolio holdings of home asset for $\alpha = 1, 5, 10$ and $\lambda = 0.1$

3.7 Conclusion

My paper provides a simple theoretical framework that illustrates how differences in investors' beliefs can generate equity home bias. In my model, all investors possess the same information about the set of possible states and the corresponding returns distribution in each state but they have different beliefs about the likelihood of these states. This heterogeneity of beliefs leads to the asymmetry of portfolio choices. This asymmetry is fundamentally different from information asymmetry in the sense that prices are not informative. The idea that investors have biased

beliefs about uncertainty of asset returns is supported by several papers in the literature on home bias.

I quantify the effect of ambiguity and ambiguity attitude on the portfolio holdings and asset prices using the stylized facts. I show that the difference in perceived uncertainty can significantly contribute to the bias towards domestic assets. The extent to which the observed bias can be explained by the differences in beliefs and ambiguity aversion depends on which parameter values one is willing to accept as reasonable. Even though ambiguity does contribute to the explanation of equity home bias, it is unlikely that the observed lack of diversification is entirely due to ambiguity aversion, which leaves room for other explanations based on institutional factors and information asymmetries. This is consistent with empirical findings that equity home bias is caused by both institutional and behavioral factors.

APPENDIX A

APPENDIX TO CHAPTER 1

A.1 Assumptions

Assumption 1a.

$$(\bar{R}_{dk} - 1) = N_k (\bar{R}_{pk} - 1) \quad (\text{A.1})$$

If there is no liquidity shock, investors are indifferent between holding direct or portfolio investment at $t = 0$.

Assumption 1b. For each country $k \in \{A, B\}$, π_{dk} should satisfy

$$\pi_{dk} + \lambda_A (1 - \pi_{dk}) > \lambda_B \quad (\text{A.2})$$

For each country k the parameters of payoff distribution have to satisfy the following assumptions:

Assumption 2a. At $t = 0$, the demand for risky asset in each country k is non-negative, i.e., $x_k^i \geq 0$ and $x_{dk}^i \geq 0$ if

$$\frac{(\bar{R}_{pk} - 1)}{\sigma_k^2/N} \geq \frac{(R_{kh} - 1)}{\sigma_k^2} \quad (\text{A.3})$$

Assumption 2b. At $t = 0$, the demand for risky asset in both countries is less than or equal to one, i.e., $\sum_{k \in \{A, B\}} x_k^i < 1$

$$\frac{(\bar{R}_{pk} - 1) - \lambda_A (\bar{R}_{pk} - p_{pk})}{(1 - \lambda_A) \gamma \sigma_k^2 / N_k} + \frac{(R_{kh} - 1) - \lambda_A (R_{kh} - p_{dk})}{(1 - \lambda_A) \gamma \sigma_k^2} < 1 \quad (\text{A.4})$$

Assumption 3. At $t = 1$, investor's demand for risky asset in both countries is less than his money holdings.

$$\sum_{k \in \{A, B\}} \max \left\{ \frac{R_{kh} - p_{dk}}{\gamma \sigma_k^2}, \frac{\bar{R}_{pk} - p_{pk}}{\gamma \sigma_k^2 / N_k} \right\} < \min \left\{ \frac{1 - x_{pk}^i}{p_{pk}}, \frac{1 - x_{dk}^i}{p_{dk}} \right\} \quad (\text{A.5})$$

where

$$\begin{aligned} x_{pk}^i &= \frac{(\bar{R}_{pk} - 1) - \lambda_A (\bar{R}_{pk} - p_{pk})}{(1 - \lambda_A) \gamma \sigma_k^2 / N_k} \\ x_{dk}^i &= \frac{(R_{kh} - 1) - \lambda_A (R_{kh} - p_{dk})}{(1 - \lambda_A) \gamma \sigma_k^2} \end{aligned} \quad (\text{A.6})$$

A.2 Investors' Decision Problem

Decision problem at $t=1$

Without loss of generality, consider the decision problem of a portfolio investor in period one. Due to the mean-variance utility and assumption 2, the demand for risky asset in period one is independent from the demand in period $t = 0$, so that direct and portfolio investors who have not received a liquidity shock have the same demands for risky asset at $t = 1$.

If at $t = 1$ a portfolio investor i chooses to buy a portfolio investment y_{pk}^i given his investment $x_{pk}^i = N_k x_k^i$ at date $t = 0$:

$$\begin{aligned} \max_{y_k} \quad & \sum_{k=a,b} \left(\begin{array}{c} 1 - x_{pk}^i - p_{pk} y_{pk} + (x_{pk}^i + y_{pk}^i) \bar{R}_{pk} \\ -\frac{1}{2} (x_{pk}^i)^2 \gamma \sigma_k^2 / N_k - \frac{1}{2} (y_{pk}^i)^2 \gamma \sigma_k^2 / N_k \end{array} \right) \\ \text{s.t.} \quad & y_{pk}^i \leq 1 - x_{pk}^i \\ & y_{pk} \geq 0 \end{aligned} \tag{A.7}$$

The optimal demand y_{pk} for portfolio investment by a portfolio investor i at country $k \in \{A, B\}$ in period $t = 1$ is given by

$$y_{pk}^i = \frac{\bar{R}_{pk} - p_{pk}}{\gamma \sigma_k^2 / N_k} \tag{A.8}$$

By assumption 2 and Property 1, the demand y_{pk}^i is interior and it does not depend on the probability of receiving a liquidity shock, so superscript i can be omitted.

Similarly, if at $t = 1$ portfolio investor i chooses to buy direct investment y_{dk}^i given his investment $N_k x_{pk}^i$ at date $t = 0$:

$$\begin{aligned} \max_{y_k} \quad & \sum_{k=a,b} \left(\begin{array}{c} 1 - x_{pk}^i - p_{dk} y_{dk} + x_{pk}^i \bar{R}_{pk} + y_{dk}^i \tilde{R}_{dk} \\ -\frac{1}{2} (x_{pk}^i)^2 \gamma \sigma_k^2 / N_k - \frac{1}{2} (y_{dk}^i)^2 \gamma \sigma_k^2 \end{array} \right) \\ \text{s.t.} \quad & p_{dk} y_{dk} \leq 1 - x_{pk}^i \\ & y_{dk} \geq 0 \end{aligned} \tag{A.9}$$

The optimal demand y_{dk} for portfolio investment by a portfolio investor i at country $k \in \{A, B\}$ in period $t = 1$ is given by

$$y_{dk}^i = \frac{\tilde{R}_{dk} - p_{dk}}{\gamma \sigma_k^2} \quad (\text{A.10})$$

Decision problem at t=0

The decision problem of a portfolio investor from country $i \in \{A, B\}$ at $t = 0$ becomes

$$\begin{aligned} \max_{x_k^i} \quad & \sum_{k=a,b} \left\{ \lambda_i (1 - N_k x_k^i + p_{pk} N_k x_k^i) + \right. \\ & \left. (1 - \lambda_i) \left(1 + N_k x_k^i (\bar{R}_{pk} - 1) - \frac{1}{2} N_k (x_k^i)^2 \gamma \bar{\sigma}_k^2 + \frac{1}{2} \frac{(\bar{R}_{pk} - p_{pk})^2}{\gamma \bar{\sigma}_k^2} \right) \right\} \\ \text{s.t.} \quad & 0 \leq x_k^i \leq 1/N_k \end{aligned} \quad (\text{A.11})$$

The optimal demand for the investment at country k by an investor from country i in period $t = 0$ is given by

$$x_k^i = \frac{(\bar{R}_{pk} - 1) - \lambda_i (\bar{R}_{pk} - p_{pk})}{(1 - \lambda_i) \gamma \bar{\sigma}_k^2} \quad (\text{A.12})$$

Then the portfolio investment is $x_{pk}^i = N_k x_k^i$.

The decision problem of a direct investor from country $i \in \{A, B\}$ at $t = 0$ becomes

$$\begin{aligned}
& \max_{x_{dk}^i} \sum_{k=a,b} \left\{ \lambda_i (1 - x_{dk}^i + p_{dk} x_{dk}^i) + \right. \\
& \quad \left. (1 - \lambda_i) \left(1 + x_{dk}^i (R_{kh} - 1) - \frac{1}{2} (x_{dk}^i)^2 \gamma \sigma_{kl}^2 + \frac{1}{2} \frac{(\bar{R}_{pk} - p_{pk})^2}{\gamma \bar{\sigma}_k^2} \right) \right\} \\
& \text{s.t.} \quad 0 \leq x_{dk}^i \leq 1
\end{aligned} \tag{A.13}$$

The optimal demand for the investment at country k by an investor from country i in period $t = 0$ is given by

$$x_{dk}^i = \frac{(R_{kh} - 1) - \lambda_i (R_{kh} - p_{dk})}{(1 - \lambda_i) \gamma \sigma_k^2} \tag{A.14}$$

A.3 Proof of Lemma 1

Proof. The optimal demand for the investment at country $k = a, b$ in period $t = 0$ is given by

$$\begin{aligned}
x_{pk}^i &= \frac{(\bar{R}_{pk} - 1) - \lambda_i (\bar{R}_{pk} - p_{pk})}{(1 - \lambda_i) \gamma \sigma_k^2 / N_k} \\
x_{dk}^i &= \frac{(R_{kh} - 1) - \lambda_i (R_{kh} - p_{dk})}{(1 - \lambda_i) \gamma \sigma_k^2}
\end{aligned} \tag{A.15}$$

First, let's show that $x_{dk}^i \geq x_{pk}^i$ for any $\lambda_i \in [\lambda_A, \lambda_B]$

$$\begin{aligned}
x_{dk}^i &= \frac{(R_{kh} - 1) - \lambda_i (R_{kh} - p_{dk})}{(1 - \lambda_i) \gamma \sigma_k^2} = \\
&= \frac{(\bar{R}_{pk} - 1) - \lambda_i (\bar{R}_{pk} - p_{dk})}{(1 - \lambda_i) \gamma \sigma_k^2 / N_k} \\
&> \frac{(\bar{R}_{pk} - 1) - \lambda_i (\bar{R}_{pk} - p_{pk})}{(1 - \lambda_i) \gamma \sigma_k^2 / N_k} = x_{pk}^i
\end{aligned}$$

The expected utilities from holding direct and portfolio investments in country k are given by

$$\begin{aligned}
EU(x_{dk}^A(\lambda_i)) &= 1 + 0.5(1 - \lambda_i) x_{dk}^2(\lambda_i) \gamma \sigma_{kl}^2 + 0.5 y_k^2 \gamma \bar{\sigma}_k^2 / N_k \\
EU(x_{pk}^A(\lambda_i)) &= 1 + 0.5(1 - \lambda_i) x_{pk}^2(\lambda_i) \gamma \bar{\sigma}_k^2 / N_k + 0.5 y_k^2 \gamma \bar{\sigma}_k^2 / N_k
\end{aligned}$$

Suppose $\delta_b > 0$, this implies that $EU(x_{dk}^A(\lambda_B)) \geq EU(x_{pk}^A(\lambda_B)) \iff x_{dk}^2(\lambda_B) \gamma \sigma_{kl}^2 \geq x_{pk}^2(\lambda_B) \gamma \bar{\sigma}_k^2$

To show that $\delta_a = 1$ we need $EU(x_{dk}^A(\lambda_A)) \geq EU(x_{pk}^A(\lambda_A)) \iff x_{dk}^2(\lambda_A) \gamma \sigma_{kl}^2 \geq x_{pk}^2(\lambda_A) \gamma \bar{\sigma}_k^2$

Taking derivative of $x_{dk}^2(\lambda_i) \gamma \sigma_{kl}^2$ and $x_{pk}^2(\lambda_i) \gamma \bar{\sigma}_k^2$ with respect to λ , we get

$$\frac{(1 - p_{dk})}{(1 - \lambda_i)^2 \gamma \sigma_{kl}^2} > \frac{(1 - p_{pk})}{(1 - \lambda_i)^2 \gamma \bar{\sigma}_k^2 / N_k}$$

The above inequality follows from

$$\frac{\tilde{R}_{dk} - p_{dk}}{\gamma \sigma_k^2} = \frac{\bar{R}_{pk} - p_{pk}}{\gamma \sigma_k^2 / N_k} \implies \frac{1 - p_{dk}}{\gamma \sigma_k^2} > \frac{1 - p_{pk}}{\gamma \sigma_k^2 / N_k}$$

Therefore, for $\lambda_A < \lambda_B$ such that $x_{dk}^2(\lambda_B) \gamma \sigma_{kl}^2 \geq x_{pk}^2(\lambda_B) \gamma \bar{\sigma}_k^2$, we have $x_{dk}^2(\lambda_A) \gamma \sigma_{kl}^2 > x_{pk}^2(\lambda_A) \gamma \bar{\sigma}_k^2$. This implies that all investors from country a obtain a higher utility by holding direct investment rather than portfolio, hence, $\delta_{Ak} = 1$.

Next, suppose $\delta_{Ak} < 1$, this implies that $EU(x_{dk}(\lambda_A)) = EU(x_{pk}(\lambda_A)) \iff x_{dk}^2(\lambda_A) \gamma \sigma_{kl}^2 = x_{pk}^2(\lambda_A) \gamma \bar{\sigma}_k^2$

$\implies x_{dk}^2(\lambda_B) \gamma \sigma_{kl}^2 < x_{pk}^2(\lambda_B) \gamma \bar{\sigma}_k^2 \iff EU(x_{dk}(\lambda_B)) < EU(x_{pk}(\lambda_B))$. Hence, $\delta_b = 0$. ■

A.4 Proof of Proposition 1

Proof. Define $\Delta EU(x_k^i) = EU(x_{dk}^i) - EU(x_{pk}^i)$ such that

$$\Delta EU(x_k^i) = x_{dk}^2(\lambda_i) \gamma \sigma_{kl}^2 - x_{pk}^2(\lambda_i) \gamma \bar{\sigma}_k^2 / N_k$$

Then the prices p_{pk} and p_{dk} are determined by equations (7) and (8). From Lemma 1 it follows that there are five possible cases that can occur in equilibrium:

Case 1. $\delta_l = 0, \delta_h = 0$ if $\Delta EU(x_k^A) < 0$

Case 2. $\delta_l \in [0, 1], \delta_h = 0$ if $\Delta EU(x_k^A) = 0$

Case 3. $\delta_l = 1, \delta_h = 0$ if $\Delta EU(x_k^A) > 0, \Delta EU(x_k^B) < 0$

Case 4. $\delta_l = 0, \delta_h \in [0, 1]$ if $\Delta EU(x_k^B) = 0$

Case 5. $\delta_l = 1, \delta_h = 1$ if $\Delta EU(x_k^B) > 0$

Part 1 (i) If $\Delta EU(x_k^a) < 0$ then by Lemma 1 $\Delta EU(x_k^b) < 0$. Therefore, there is no direct investment in the equilibrium, i.e., $\delta_{Ak} = 0, \delta_{Bk} = 0$. The condition $\Delta EU(x_k^a) < 0$ implies that

$$\frac{\alpha \frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + (1-\alpha) \frac{\lambda_{Ak}\lambda_{Bk}}{(1-\lambda_{Bk})}}{\alpha \frac{\lambda_{Ak}+(1-\lambda_{Ak})^2}{(1-\lambda_{Ak})} + (1-\alpha) \frac{\lambda_{Bk}+(1-\lambda_{Bk})^2}{(1-\lambda_{Bk})}} < \frac{\left(\frac{\sigma_{kl}^2}{\bar{\sigma}_k^2/N_k}\right)^{0.5} - N_k}{\left(\frac{\sigma_{kl}^2}{\bar{\sigma}_k^2/N_k}\right)^{0.5} - \frac{\tilde{\sigma}_k^2}{\bar{\sigma}_k^2/N_k}}.$$

The equilibrium prices are given by

$$\begin{aligned} p_{pk} &= \bar{R}_{pk} - \left(\frac{\alpha \frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + (1-\alpha) \frac{\lambda_{Bk}}{(1-\lambda_{Bk})}}{\alpha \frac{\lambda_{Ak}+(1-\lambda_{Ak})^2}{(1-\lambda_{Ak})} + (1-\alpha) \frac{\lambda_{Bk}+(1-\lambda_{Bk})^2}{(1-\lambda_{Bk})}} \right) (\bar{R}_{pk} - 1), \\ p_{dk} &= \bar{R}_{dk} - \frac{\tilde{\sigma}_k^2}{\bar{\sigma}_k^2/N_k} \left(\frac{\alpha \frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + (1-\alpha) \frac{\lambda_{Bk}}{(1-\lambda_{Bk})}}{\alpha \frac{\lambda_{Ak}+(1-\lambda_{Ak})^2}{(1-\lambda_{Ak})} + (1-\alpha) \frac{\lambda_{Bk}+(1-\lambda_{Bk})^2}{(1-\lambda_{Bk})}} \right) (\bar{R}_{pk} - 1), \end{aligned}$$

and $(x_{dk}^i, x_{pk}^i, y_k)$ are given by (17) and (10).

(ii) Next $\Delta EU(x_k^a) = 0$ then together with Property 2., we can derive the equilibrium prices:

$$\begin{aligned} p_{pk} &= \bar{R}_{pk} - \frac{1}{\lambda_{Ak}} \left(\frac{\left(\frac{\sigma_{kl}^2}{\bar{\sigma}_k^2/N_k}\right)^{0.5} - N_k}{\left(\frac{\sigma_{kl}^2}{\bar{\sigma}_k^2/N_k}\right)^{0.5} - \frac{\tilde{\sigma}_k^2}{\bar{\sigma}_k^2/N_k}} \right) (\bar{R}_{pk} - 1) \\ p_{dk} &= \bar{R}_{dk} - \frac{1}{\lambda_{Ak}} \frac{\tilde{\sigma}_k^2}{\bar{\sigma}_k^2/N_k} \left(\frac{\left(\frac{\sigma_{kl}^2}{\bar{\sigma}_k^2/N_k}\right)^{0.5} - N_k}{\left(\frac{\sigma_{kl}^2}{\bar{\sigma}_k^2/N_k}\right)^{0.5} - \frac{\tilde{\sigma}_k^2}{\bar{\sigma}_k^2/N_k}} \right) (\bar{R}_{pk} - 1) \end{aligned}$$

Then δ_{Ak} is determined by market clearing condition:

$$\delta_{Ak} = \frac{(\alpha(1-\lambda_A) + (1-\alpha)(1-\lambda_A))y_k - \alpha\lambda_A x_{pk}(\lambda_A) + (1-\alpha)\lambda_B x_{pk}(\lambda_B)}{\alpha(\lambda_A + (1-\lambda_A)\pi_k)x_{dk}(\lambda_A) - \alpha\lambda_A x_{pk}(\lambda_A)}$$

If $\Delta EU(x_k^a) \geq 0$ then $\delta_{Ak} \geq 0$. If $\Delta EU(x_k^a) = 0$ and $\delta_{Ak} \leq 1$ then by Lemma 1 $\Delta EU(x_k^b) > 0$ which implies that $\delta_{Bk} = 0$. Case 1 and 2 constitute type I equilibrium. If type I equilibrium exist, it is unique.

Part 2. Next consider $\Delta EU(x_k^a) > 0$ and $\delta_{Ak} \geq 1$ then $\Delta EU(x_k^b)$ can be less than, equal to, or greater than zero.

(iii) Consider $\Delta EU(x_k^a) > 0, \delta_{Ak} \geq 1$ and $\Delta EU(x_k^b) < 0$. Then $\delta_{Bk} = 0$. This is a Case 3: separating equilibrium with $\delta_{Ak} = 1$ and $\delta_{Bk} = 0$.

$$\begin{aligned}
\overline{R}_{pk} - (\overline{R}_p - 1) \\
p_{pk} &= \left(\frac{\alpha \left(\frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + \pi_k \right) \frac{\sigma_k^2}{\sigma_{kl}^2} + (1-\alpha) \frac{\lambda_{Bk}}{(1-\lambda_{Bk})}}{\alpha(1-\lambda_{Ak}) + \alpha\lambda_{Ak} \left(\frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + \pi_k \right) \frac{\sigma_k^2}{\sigma_{kl}^2} + (1-\alpha) \frac{\lambda_{Bk} + (1-\lambda_{Bk})^2}{(1-\lambda_{Bk})}} \right) \\
\overline{R}_{dk} - \frac{\sigma_k^2}{\sigma_{kl}^2/N_k} (\overline{R}_p - 1) \\
p_{dk} &= \left(\frac{\alpha \left(\frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + \pi_k \right) \frac{\sigma_k^2}{\sigma_{kl}^2} + (1-\alpha) \frac{\lambda_{Bk}}{(1-\lambda_{Bk})}}{\alpha(1-\lambda_{Ak}) + \alpha\lambda_{Ak} \left(\frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + \pi_k \right) \frac{\sigma_k^2}{\sigma_{kl}^2} + (1-\alpha) \frac{\lambda_{Bk} + (1-\lambda_{Bk})^2}{(1-\lambda_{Bk})}} \right)
\end{aligned}$$

such that

$$\begin{aligned}
& \frac{1}{\lambda_{Ak}} \frac{\left(\frac{\sigma_{kl}^2}{\sigma_k^2/N_k} \right)^{0.5} - N_k}{\left(\frac{\sigma_{kl}^2}{\sigma_k^2/N_k} \right)^{0.5} - \frac{\sigma_k^2}{\sigma_{kl}^2/N_k}} < \\
& < \frac{\alpha \left(\frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + \pi_k \right) \frac{\sigma_k^2}{\sigma_{kl}^2} + (1-\alpha) \frac{\lambda_{Bk}}{(1-\lambda_{Bk})}}{\alpha(1-\lambda_{Ak}) + \alpha\lambda_{Ak} \left(\frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + \pi_k \right) \frac{\sigma_k^2}{\sigma_{kl}^2} + (1-\alpha) \frac{\lambda_{Bk} + (1-\lambda_{Bk})^2}{(1-\lambda_{Bk})}} \\
& < \frac{1}{\lambda_{Bk}} \frac{\left(\frac{\sigma_{kl}^2}{\sigma_k^2/N_k} \right)^{0.5} - N_k}{\left(\frac{\sigma_{kl}^2}{\sigma_k^2/N_k} \right)^{0.5} - \frac{\sigma_k^2}{\sigma_{kl}^2/N_k}}
\end{aligned}$$

(iv) Consider $\Delta EU(x_k^a) > 0, \delta_{Ak} \geq 1$ and $\Delta EU(x_k^b) = 0$. Then prices are

given by

$$\begin{aligned}
p_{pk} &= \bar{R}_{pk} - \frac{1}{\lambda_B} \frac{\left(\frac{\sigma_{kl}^2}{\bar{\sigma}_k^2/N_k}\right)^{0.5} - N_k}{\left(\frac{\sigma_{kl}^2}{\bar{\sigma}_k^2/N_k}\right)^{0.5} - \frac{\tilde{\sigma}_k^2}{\bar{\sigma}_k^2/N_k}} (\bar{R}_{pk} - 1) \\
p_{dk} &= \bar{R}_{dk} - \frac{\tilde{\sigma}_k^2}{\bar{\sigma}_k^2/N_k} \frac{1}{\lambda_B} \frac{\left(\frac{\sigma_{kl}^2}{\bar{\sigma}_k^2/N_k}\right)^{0.5} - N_k}{\left(\frac{\sigma_{kl}^2}{\bar{\sigma}_k^2/N_k}\right)^{0.5} - \frac{\tilde{\sigma}_k^2}{\bar{\sigma}_k^2/N_k}} (\bar{R}_{pk} - 1)
\end{aligned}$$

The equilibrium fraction of direct investors from country b is determined by market clearing condition. Contrary to the Part 1, the market clearing condition is no longer linear in δ_{Bk} since market beliefs about the probability of direct investor receiving a liquidity shock (λ_d) depends on δ_{Bk} ($\lambda_d = \frac{\alpha\delta_{Ak}\lambda_A + (1-\alpha)\delta_{Bk}\lambda_B}{\alpha\delta_{Ak} + (1-\alpha)\delta_{Bk}}$), and therefore, variance $\tilde{\sigma}_k^2$ also depends on δ_{Bk} . The equilibrium δ_{Bk} is determined by the market clearing condition

$$\begin{aligned}
&\alpha (\lambda_A + (1 - \lambda_A) \pi_{kk}) x_{dk} (\lambda_A) + \\
&+ (1 - \alpha) (\lambda_B + (1 - \lambda_B) \pi_k) \delta_{Bk} x_{dk} (\lambda_B) \\
&+ (1 - \alpha) \lambda_B (1 - \delta_{Bk}) x_{pk} (\lambda_B) \\
&= [\alpha (1 - \lambda_A) + (1 - \alpha) (1 - \lambda_B)] y
\end{aligned}$$

We can write excess demand as a quadratic equation in δ_{Bk} : $ED = c_1\delta_{Bk}^2 + c_2\delta_{Bk} + c_3$ where

$$C_1 = -(1 - \alpha)^2 \frac{\pi_k (1 - \pi_k) (\sigma_{kh}^2 - \sigma_{kl}^2)}{(1 - \pi_k) \sigma_{kl}^2 + \pi_k \sigma_{kh}^2} \begin{pmatrix} \lambda_{Bk} \left(\left(\frac{\bar{\sigma}_k^2}{N_k \sigma_{kl}^2} \right)^{0.5} - 1 \right) \\ + \pi_k (1 - \lambda_{Bk}) \left(\frac{\bar{\sigma}_k^2}{N_k \sigma_{kl}^2} \right)^{0.5} \end{pmatrix}$$

$$C_2 = \left(\begin{array}{l} \alpha \left(\frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + \pi_k \right) \frac{\lambda_{Ak}}{\lambda_{Bk}} \left(\left(\frac{\sigma_{kl}^2}{N_k \bar{\sigma}_k^2} \right)^{0.5} - 1 \right) (1-\alpha) (1 + (1-\pi_k) \lambda_{Bk}) \\ - \alpha \left(\frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + \pi_k \right) \left(\left(\frac{\bar{\sigma}_k^2}{N_k \sigma_{kl}^2} \right)^{0.5} - 1 \right) (1-\alpha) (1 + (1-\pi_k) \lambda_{Bk}) \\ + (\alpha (1-\lambda_{Ak}) + (1-\alpha) (1-\lambda_{Bk})) \frac{1}{\lambda_{Bk}} \left(\left(\frac{\sigma_{kl}^2}{N_k \bar{\sigma}_k^2} \right)^{0.5} - 1 \right) \\ (1-\alpha) (1 + (1-\pi_k) \lambda_{Bk}) \\ + \alpha \left(\frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + \pi_k \right) \frac{\pi_k (1-\pi_k) (\sigma_{kh}^2 - \sigma_{kl}^2)}{(1-\pi_k) \sigma_{kl}^2 + \pi_k \sigma_{kh}^2} (1-\alpha) (1-\lambda_{Bk}) \\ + \left(\frac{\lambda_{Ak}}{\lambda_{Bk}} \left(\left(\frac{\sigma_{kl}^2}{N_k \bar{\sigma}_k^2} \right)^{0.5} - 1 \right) + 1 \right) \\ + (1-\alpha) \left(\frac{\bar{\sigma}_k^2}{N_k \sigma_{kl}^2} \right)^{0.5} \pi_k \frac{\pi_k (1-\pi_k) (\sigma_{kh}^2 - \sigma_{kl}^2)}{(1-\pi_k) \sigma_{kl}^2 + \pi_k \sigma_{kh}^2} \alpha (1-\lambda_{Ak}) \\ + (1-\alpha) \frac{\lambda_{Bk}}{(1-\lambda_{Bk})} \left(\left(\frac{\bar{\sigma}_k^2}{N_k \sigma_{kl}^2} \right)^{0.5} - 1 \right) \frac{\pi_k (1-\pi_k) (\sigma_{kh}^2 - \sigma_{kl}^2)}{(1-\pi_k) \sigma_{kl}^2 + \pi_k \sigma_{kh}^2} \alpha (1-\lambda_{Ak}) \\ (1-\alpha) \frac{\lambda_{Bk}}{(1-\lambda_{Bk})} \frac{\pi_k (1-\pi_k) (\sigma_{kh}^2 - \sigma_{kl}^2)}{(1-\pi_k) \sigma_{kl}^2 + \pi_k \sigma_{kh}^2} [(1-\alpha) (1-\lambda_{Bk})] \end{array} \right)$$

$$C_3 = \left(\begin{array}{l} -\alpha (\alpha + (1-\pi_k) \alpha \lambda_{Ak}) \left(\frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + \pi_k \right) \\ \left(\left(\left(\frac{\bar{\sigma}_k^2}{N_k \sigma_{kl}^2} \right)^{0.5} - 1 \right) - \frac{\lambda_{Ak}}{\lambda_{Bk}} \left(\left(\frac{\sigma_{kl}^2}{N_k \bar{\sigma}_k^2} \right)^{0.5} - 1 \right) \right) \\ + (\alpha (1-\lambda_{Ak}) + (1-\alpha) (1-\lambda_{Bk})) \frac{1}{\lambda_{Bk}} \\ (\alpha + (1-\pi_k) \alpha \lambda_{Ak}) \left(\left(\frac{\sigma_{kl}^2}{N_k \bar{\sigma}_k^2} \right)^{0.5} - 1 \right) \\ - \alpha \left(\frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + \pi_k \right) \frac{\lambda_{Ak}}{\lambda_{Bk}} \left(\left(\frac{\sigma_{kl}^2}{N_k \bar{\sigma}_k^2} \right)^{0.5} - 1 \right) \frac{\pi_k (1-\pi_k) (\sigma_{kh}^2 - \sigma_{kl}^2)}{(1-\pi_k) \sigma_{kl}^2 + \pi_k \sigma_{kh}^2} \alpha (1-\lambda_{Ak}) \\ \left[\alpha \left(\frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + \pi_k \right) + (1-\alpha) \frac{\lambda_{Bk}}{(1-\lambda_{Bk})} \right] \frac{\pi_k (1-\pi_k) (\sigma_{kh}^2 - \sigma_{kl}^2)}{(1-\pi_k) \sigma_{kl}^2 + \pi_k \sigma_{kh}^2} \alpha (1-\lambda_{Ak}) \end{array} \right)$$

$C_1 < 0$ and $\max_{\delta_{Bk}} ED > 0$ (follows from $\Delta EU(x_k^A) > 0$). If $C_2 > 0$ and $C_3 < 0$ then there are 2 interior $\delta_{Bk} \in (0, 1)$

If there are two equilibria with $\delta_{Ak} = 1, \delta_{Bk} \in (0, 1]$ such that $\Delta EU(x_k^b) = 0$ and $\Delta EU(x_k^a) > 0$ then at $\delta_{Bk} = 0$, $ED < 0$ which implies that

Case 3 (separating equilibrium) is also an equilibrium with $\delta_{Bk} = 0, \delta_{Ak} = 1$. If $C_2 > 0$ and $C_3 > 0$ the the equilibrium is unique. The existence of the unique root follows from $ED \leq 0$ at $\delta_{Bk} = 1$. If $C_2 < 0$ then $C_3 < 0$ which implies the unique solution.

(v) Next consider $\Delta EU(x_k^a) > 0, \delta_{Ak} \geq 1$ and $\Delta EU(x_k^b) > 0$. This is a case 5 equilibrium with $\delta_{Bk}^* = 1, \delta_{Ak}^* = 1$ and prices given by

$$(\bar{R}_p - p) = \frac{(\bar{R}_p - 1) \left(\alpha \left(\frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + \pi_k \right) + (1-\alpha) \left(\frac{\lambda_{Bk}}{(1-\lambda_{Bk})} + \pi_k \right) \right) \frac{\bar{\sigma}_k^2}{\sigma_{kl}^2}}{\alpha(1-\lambda_{Ak}) + (1-\alpha)(1-\lambda_{Bk}) + \left(\alpha\lambda_{Ak} \left(\frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + \pi_k \right) + (1-\alpha)\lambda_{Bk} \left(\frac{\lambda_{Bk}}{(1-\lambda_{Bk})} + \pi_k \right) \right) \frac{\tilde{\sigma}_k^2}{\sigma_{kl}^2}},$$

such that

$$\left(\frac{\lambda_B \left(\alpha \left(\frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + \pi_k \right) + (1-\alpha) \left(\frac{\lambda_{Bk}}{(1-\lambda_{Bk})} + \pi_k \right) \right) \frac{\bar{\sigma}_k^2}{\sigma_{kl}^2}}{\alpha(1-\lambda_{Ak}) + (1-\alpha)(1-\lambda_{Bk}) + \left(\alpha\lambda_{Ak} \left(\frac{\lambda_{Ak}}{(1-\lambda_{Ak})} + \pi_k \right) + (1-\alpha)\lambda_{Bk} \left(\frac{\lambda_{Bk}}{(1-\lambda_{Bk})} + \pi_k \right) \right) \frac{\tilde{\sigma}_k^2}{\sigma_{kl}^2}} \right) > \frac{\left(\frac{\sigma_{kl}^2}{\sigma_k^2/N_k} \right)^{0.5} - N_k}{\left(\frac{\sigma_{kl}^2}{\sigma_k^2/N_k} \right)^{0.5} - \frac{\bar{\sigma}_k^2}{\sigma_k^2/N_k}}$$

All three cases are captured by type II equilibria and can be summarized in the following way: If $\Delta EU(x_k^a) > 0$ and $\delta_{Ak} \geq 1$ and

◦ at $\delta_{Bk} = 0 : \Delta EU(x_k^b) < 0$ then there is at least one equilibrium with $\delta_{Bk} = 0, \delta_{Ak} = 1$:

◦ at $\delta_{Bk} = 0 : \Delta EU(x_k^b) < 0$ and at $\delta_{Bk} = 1 : \Delta EU(x_k^b) > 0$ then there is 2 equilibria

◦ at $\delta_{Bk} = 0 : \Delta EU(x_k^b) < 0$ and at $\delta_{Bk} = 1 : \Delta EU(x_k^b) < 0$ and $\max_{\delta_{Bk}} \{\Delta EU(x_k^b)\} > 0$ then there is 3 equilibria

◦ at $\delta_{Bk} = 0 : \Delta EU(x_k^b) < 0$ and at $\delta_{Bk} = 1 : \Delta EU(x_k^b) < 0$ and $\max_{\delta_{Bk}} \{\Delta EU(x_k^b)\} = 0$ then there is 2 equilibria

◦ at $\delta_{Bk} = 0 : \Delta EU(x_k^b) < 0$ and at $\delta_{Bk} = 1 : \Delta EU(x_k^b) > 0$ then there is 2 equilibria

◦ at $\delta_{Bk} = 0 : \Delta EU(x_k^b) < 0$ and at $\delta_{Bk} = 1 : \Delta EU(x_k^b) < 0$ and $\max_{\delta_{Bk}} \{\Delta EU(x_k^b)\} < 0$ then there is 1 equilibrium

◦ at $\delta_{Bk} = 0 : \Delta EU(x_k^b) > 0$ and at $\delta_{Bk} = 1 : \Delta EU(x_k^b) < 0$ then there is 1 equilibrium

◦ at $\delta_{Bk} = 0 : \Delta EU(x_k^b) > 0$ and at $\delta_{Bk} = 1 : \Delta EU(x_k^b) > 0$ then there is no equilibrium ■

A.5 Proof of Property 4

Proof. Consider $\Delta x_k^i = x_{dk}(\lambda_i) - x_{pk}(\lambda_i)$

$$\begin{aligned}
\Delta x_k^i &= \frac{\left((\bar{R}_{pk} - 1) - \lambda_i \frac{\tilde{\sigma}_k^2}{\bar{\sigma}_k^2} (\bar{R}_{pk} - p_{pk}) \right)^2}{(1 - \lambda_i) \gamma \bar{\sigma}_{kl}^2 / N_k} - \frac{\left((\bar{R}_{pk} - 1) - \lambda_i (\bar{R}_{pk} - p_{pk}) \right)^2}{\bar{\sigma}_k^2 / N_k} \\
&= \frac{(\bar{R}_{pk} - 1)}{(1 - \lambda_i) \gamma \bar{\sigma}_k^2 / N_k} \left[\frac{\bar{\sigma}_k^2}{\sigma_{kl}^2} - 1 \right] - \frac{\lambda_i}{(1 - \lambda_i)} \frac{(\bar{R}_{pk} - p_{pk})}{\bar{\sigma}_k^2 / N_k} \left[\frac{\tilde{\sigma}_k^2}{\sigma_{kl}^2} - 1 \right] \\
&= \left(\begin{aligned} &\frac{(\bar{R}_{pk} - 1)}{(1 - \lambda_i) \gamma \bar{\sigma}_k^2 / N_k} \pi_k \left(\frac{\sigma_{kh}^2}{\sigma_{kl}^2} - 1 \right) \\ &- \frac{\lambda_i}{(1 - \lambda_i)} \frac{(\bar{R}_{pk} - p_{pk})}{\gamma \bar{\sigma}_k^2 / N_k} \frac{\pi_k}{\pi_k + \lambda_{dk}(1 - \pi_k)} \left(\frac{\sigma_{kh}^2}{\sigma_{kl}^2} - 1 \right) \end{aligned} \right)
\end{aligned}$$

Δx_k^i is increasing function of $\frac{\sigma_{kh}^2 - \sigma_{kl}^2}{\bar{\sigma}_k^2}$ for any of the two pooling type of the equilibrium prices. It can be shown that $\Delta EU(x_k^a)$ is also increasing function in $\frac{\sigma_{kh}^2 - \sigma_{kl}^2}{\bar{\sigma}_k^2}$. Therefore, if $\frac{\sigma_{kh}^2 - \sigma_{kl}^2}{\bar{\sigma}_k^2}$ increases than δ_k also increases .

If $(\bar{R}_{dk} - \bar{R}_{pk})$ are larger and/or N_k are smaller then δ_k is larger in the equilibrium. ■

A.6 Proof of Proposition 2

Proof. (1) consider λ_A as a host country. The type II pooling equilibrium should satisfy the following conditions:

$$\begin{aligned} p_{pk} &= \bar{R}_{pk} - \frac{1}{\lambda_B} \frac{\left(\frac{\sigma_{kl}^2}{\bar{\sigma}_k^2/N_k}\right)^{0.5} - N_k}{\left(\frac{\sigma_{kl}^2}{\bar{\sigma}_k^2/N_k}\right)^{0.5} - \frac{\tilde{\sigma}_k^2}{\bar{\sigma}_k^2/N_k}} (\bar{R}_{pk} - 1) \\ p_{dk} &= \bar{R}_{dk} - \frac{\tilde{\sigma}_k^2}{\bar{\sigma}_k^2/N_k} \frac{1}{\lambda_B} \frac{\left(\frac{\sigma_{kl}^2}{\bar{\sigma}_k^2/N_k}\right)^{0.5} - N_k}{\left(\frac{\sigma_{kl}^2}{\bar{\sigma}_k^2/N_k}\right)^{0.5} - \frac{\tilde{\sigma}_k^2}{\bar{\sigma}_k^2/N_k}} (\bar{R}_{pk} - 1) \end{aligned}$$

And δ_{Bk} is determined from

$$\begin{aligned} &\alpha (\lambda_A + (1 - \lambda_A) \pi_{kk}) x_{dk}(\lambda_A) \\ &+ (1 - \alpha) (\lambda_B + (1 - \lambda_B) \pi_k) \delta_{Bk} x_{dk}(\lambda_B) \\ &+ (1 - \alpha) \lambda_B (1 - \delta_{Bk}) x_{pk}(\lambda_B) \\ &= (\alpha (1 - \lambda_A) + (1 - \alpha) (1 - \lambda_B)) y_k \end{aligned}$$

Define excess demand by ED . We can write the market clearing condition as a quadratic equation in δ_{Bk} : $ED = c_1 \delta_{Bk}^2 + c_2 \delta_{Bk} + c_3 = 0$ where $c_1 < 0$. There are 2 possibilities: either unique equilibrium or two equilibria. There are

two equilibria if $c_3 < 0$. If λ_A increases to λ'_A then the $\max_{\delta_{Bk}} ED$ increases and $\arg \max_{\delta_{Bk}} ED$ decreases. Denote δ_{Bk}^* and δ_{Bk}^{**} the two solutions to $ED = 0$ such that $\delta_{Bk}^* < \delta_{Bk}^{**}$. So that $\delta_{Bk}^*(\lambda_A) < \delta_{Bk}^*(\lambda'_A)$ and $\delta_{Bk}^{**}(\lambda_A) > \delta_{Bk}^{**}(\lambda'_A)$. If there is a unique equilibrium ($c_3 > 0$) then only the solution δ_{Bk}^{**} remains. Therefore, if equilibrium is unique then the increase in λ_A leads to a higher fraction of direct investors in equilibrium. If there are multiple equilibria, then the effect is ambiguous.

(2) consider λ_B as a host country. If λ_B increases to λ_B then the $\max_{\delta_{Bk}} ED$ decreases and $\arg \max_{\delta_{Bk}} ED$ increases. In this case $\delta_{Ak}^*(\lambda_B) > \delta_{Ak}^*(\lambda'_B)$ and $\delta_{Ak}^{**}(\lambda_B) < \delta_{Ak}^{**}(\lambda'_B)$. If there is a unique equilibrium ($c_3 > 0$) then only the solution δ_{Bk}^* remains. Therefore, if equilibrium is unique then the increase in λ_B leads to a higher fraction of direct investors in equilibrium. If there are multiple equilibria, then the effect is ambiguous depending on the equilibrium. ■

APPENDIX B
APPENDIX TO CHAPTER 2

B.1 Private Information Equilibrium

B.1.1 Proof of Proposition 1

Proof. The market clearing in state s is given by $\lambda x p_s = (1 - \lambda)(1 - x)$. Therefore, $p_1 = p_2 = p$ since x is decided at $t = 0$. Hence, an investor's maximization problem becomes

$$EU_s(x, p) = \lambda \log(1 - x + px) + (1 - \lambda) \sum_{s=1,2} q_s \left(\begin{aligned} &\pi_s \log(xr + (1 - x)\bar{R}_s/p) \\ &+ (1 - \pi_s) \log(xR_H + (1 - x)\bar{R}_s/p) \end{aligned} \right)$$

The equilibrium price and investment allocation (x, p) are determined by the following system of equations:

$$\begin{aligned} \lambda \frac{p-1}{x(p-1)+1} + (1 - \lambda) \sum_{s=1,2} q_s \left(\pi_s \frac{r - \bar{R}_s/p}{x(r - \bar{R}_s/p) + \bar{R}_s/p} + (1 - \pi_s) \frac{R_H - \bar{R}_s/p}{x(R_H - \bar{R}_s/p) + \bar{R}_s/p} \right) &= 0 \\ \lambda xp - (1 - \lambda)(1 - x) &= 0 \end{aligned}$$

Therefore, the equilibrium price is given by

$$p_a^* = \frac{\lambda + \sum_{s=1,2} q_s \left(\pi_s \frac{\bar{R}_s}{r + \bar{R}_s \frac{\lambda}{(1-\lambda)}} + (1 - \pi_s) \frac{\bar{R}_s}{R_H + \bar{R}_s \frac{\lambda}{(1-\lambda)}} \right)}{\lambda + \sum_{s=1,2} q_s \left(\pi_s \frac{r}{r + \bar{R}_s \frac{\lambda}{(1-\lambda)}} + (1 - \pi_s) \frac{R_H}{R_H + \bar{R}_s \frac{\lambda}{(1-\lambda)}} \right)}$$

By assumption 3 and 4, the equilibrium price p satisfies the dynamic consistency conditions. Assumption 3 rules out the situation that a risky asset dominates the safe asset at $t = 1$. If the market price $p \geq 1$, then no one will choose to hold the safe asset at $t = 0$. Assumption 4 rules out the situation that the safe

asset dominates a risky asset at $t = 1$. If the market price $p < p(\bar{r})$ such that $EU(p(\bar{r}), x(\bar{r})) = (1 - \lambda) \sum_{s=1,2} q_s \log(\bar{R}_s/p(\bar{r}))$ then the return on the risky asset bought at $t = 1$ is higher than the return on investment made at $t = 0$, hence, no one will choose to invest in risky projects at $t = 0$.

If the market price $p \geq r$ then the equilibrium investment allocation x is given by

$$x_a^* = (1 - \lambda) \left(\lambda + \sum_{s=1,2} q_s \left(\pi_s \frac{r}{\left(r + \bar{R}_s \frac{\lambda}{1-\lambda}\right)} + (1 - \pi_s) \frac{R_H}{\left(R_H + \bar{R}_s \frac{\lambda}{1-\lambda}\right)} \right) \right)$$

If the market price $p < r$ then an investor's maximization problem becomes

$$EU_{no\ trade}(x) = \left(\begin{array}{c} \lambda \log(1 - x + rx) \\ + (1 - \lambda) \sum_{s=1,2} q_s \left(\begin{array}{c} \pi_s \log(xr + (1 - x)) \\ + (1 - \pi_s) \log(xR_H + (1 - x)) \end{array} \right) \end{array} \right)$$

Therefore, the equilibrium investment allocation x is given by

$$x_a^{**} = \frac{(\lambda + (1 - \lambda) \bar{\pi})(r - 1) + (1 - \lambda)(1 - \bar{\pi})(R_H - 1)}{(1 - r)(R_H - 1)}$$

In both cases, the corner solutions: $x=0$ and $x=1$ are dominated by the interior solution. If all endowments are invested in risky assets: $x = 1$, then the consumption at date 1 $c_1 = 0$, which implies the utility equal to negative infinity. If all endowment is kept in the safe asset then the expected utility is zero while interior solution yields the positive utility since $\bar{R}_s > 1$.

If it exists, the market equilibrium always dominates the no trade equilibrium since it provides a higher consumption in each state in both dates. Suppose not, let x_a^{**} be a solution to the investor maximization problem even if $p \geq r$. The expected utility in the market equilibrium is larger than the $EU_{market}(x_a^{**}, p) >$

$EU_{no\ trade}(x_a^{**})$ since $\bar{R}_s/p > 1$ and $p \geq r$,

$$\begin{aligned}
& EU_{market}(x_a^{**}, p) \\
& \quad \lambda \log(1 - x_a^{**} + px_a^{**}) \\
& = \quad + (1 - \lambda) \sum_{s=1,2} q_s \left(\begin{aligned} & \pi_s \log(x_a^{**}p + (1 - x_a^{**})\bar{R}_s/p) \\ & + (1 - \pi_s) \log(x_a^{**}R_H + (1 - x_a^{**})\bar{R}_s/p) \end{aligned} \right) \\
& \quad \lambda \log(1 - x_a^{**} + rx_a^{**}) \\
& > \quad + (1 - \lambda) \sum_{s=1,2} q_s \left(\begin{aligned} & \pi_s \log(x_a^{**}r + (1 - x_a^{**})) \\ & + (1 - \pi_s) \log(x_a^{**}R_H + (1 - x_a^{**})) \end{aligned} \right) \\
& = EU_{no\ trade}(x_a^{**})
\end{aligned}$$

and $\forall x : EU_{market}(x_a^*, p) \geq EU_{market}(x, p)$. Contradiction. It is impossible to have market equilibrium in one state and no trade equilibrium in another state Since the market price is the same in both states.

Furthermore, the investment allocation is larger in the market equilibrium relative to no trade equilibrium: $x_a^* > x_a^{**}$

$$\begin{aligned}
x_a^* - x_a^{**} & = \quad \lambda \left((1 - \lambda) + \frac{1}{(R_H - 1)} \right) \\
& \quad + (1 - \lambda) \sum_{s=1,2} q_s \left(\begin{aligned} & \pi_s \left(\frac{r}{(r + \bar{R}_s \frac{\lambda}{1 - \lambda})} + \frac{1}{(R_H - 1)} \right) \\ & + (1 - \pi_s) \left(\frac{R_H}{(R_H + \bar{R}_s \frac{\lambda}{1 - \lambda})} - \frac{1}{(1 - r)} \right) \end{aligned} \right) \\
& = \quad \lambda \left((1 - \lambda) + \frac{1}{(R_H - 1)} \right) \\
& \quad + (1 - \lambda) \sum_{s=1,2} q_s \left(\pi_s \left(\frac{R_H r + \bar{R}_s \frac{\lambda}{1 - \lambda}}{r + \bar{R}_s \frac{\lambda}{1 - \lambda}} \right) + (1 - \pi_s) \left(\frac{R_H r + \bar{R}_s \frac{\lambda}{1 - \lambda}}{R_H + \bar{R}_s \frac{\lambda}{1 - \lambda}} \right) \right) > 0
\end{aligned}$$

The market equilibrium consumption:

$$\begin{aligned}
c_1 & = \frac{(1 - x)}{\lambda} \\
c_1 & = \left(\lambda + (1 - \lambda) \sum_{s=1,2} q_s \left(\pi_s \frac{\bar{R}_s}{(1 - \lambda)r + \lambda \bar{R}_s} + (1 - \pi_s) \frac{\bar{R}_s}{(1 - \lambda)R_H + \lambda \bar{R}_s} \right) \right)
\end{aligned}$$

$$\begin{aligned}
c_{2i}(s) &= x \left(R_i + \bar{R}_s \frac{\lambda}{1-\lambda} \right) \\
c_{2i}(s) &= \frac{(1-\lambda) \left(R_i + \frac{\lambda}{1-\lambda} \bar{R}_s \right)}{\left(\lambda + \sum_{s=1,2} q_s \left(\pi_s \frac{r}{(r + \bar{R}_s \frac{\lambda}{1-\lambda})} + (1-\pi_s) \frac{R_H}{(R_H + \bar{R}_s \frac{\lambda}{1-\lambda})} \right) \right)}
\end{aligned}$$

Note, $c_1 \leq 1$ since $\sum_{s=1,2} q_s \left(\pi_s \frac{(1-\lambda)(\bar{R}_s - r)}{(1-\lambda)r + \lambda \bar{R}_s} + (1-\pi_s) \frac{(1-\lambda)(\bar{R}_s - R_H)}{(1-\lambda)R_H + \lambda \bar{R}_s} \right) \leq 0$ which is implied by $p \leq 1$.

The no trade equilibrium consumption:

$$\begin{aligned}
c_1 &= 1 - x + rx \\
c_1 &= (\lambda + (1-\lambda)\bar{\pi}) \frac{R_H - r}{R_H - 1}
\end{aligned}$$

$$\begin{aligned}
c_{2i}(s) &= xR_i + (1-x) \\
c_{2H}(s) &= (1-\lambda)(1-\bar{\pi}) \frac{(R_H - r)}{(1-r)} \\
c_{2L}(s) &= (\lambda + (1-\lambda)\bar{\pi}) \frac{R_H - r}{R_H - 1}
\end{aligned}$$

■

B.2 Equilibrium with Adverse Selection

B.2.1 Proof of Proposition 2

Proof. Similarly to equilibrium without adverse selection, if the market equilibrium exist in a state s then it will dominate an equilibrium with no trade. Consider

type (1) equilibrium:

$$\begin{aligned} \max_x \quad & \lambda \log(1 - x + p_s x) + (1 - \lambda) \sum_{s=1,2} q_s \left(\begin{aligned} & \pi_s \log \left(x p_s + (1 - x) \tilde{R}_s / p_s \right) \\ & + (1 - \pi_s) \log \left(x R_H + (1 - x) \tilde{R}_s / p_s \right) \end{aligned} \right) \\ \text{s.t} \quad & (i) \quad 0 \leq x \leq 1 \\ & (ii) \quad p_s \geq r \quad \forall s \end{aligned}$$

Therefore, the type 1 equilibrium investment allocation and market prices are determined by the following equations:

$$\begin{aligned} \sum_{s=1,2} q_s \left(\begin{aligned} & \lambda \frac{p_s - 1}{1 - x + p_s x} + \\ & + (1 - \lambda) \left(\pi \frac{p_s - \tilde{R}_s / p_s}{x p_s + (1 - x) \tilde{R}_s / p_s} + (1 - \pi) \frac{R_H - \tilde{R}_s / p_s}{x R_H + (1 - x) \tilde{R}_s / p_s} \right) \end{aligned} \right) = 0 \\ (\lambda + (1 - \lambda) \pi_s) p_s x = (1 - \lambda) (1 - x) \end{aligned}$$

Substituting prices p_s , we can get

$$F_b(x) \equiv \sum_{s=1,2} q_s \left(\begin{aligned} & \lambda \frac{1}{\left(\frac{1}{(1-\lambda)} + \pi_s\right)} \\ & + (1 - \lambda) \pi \frac{(1-x)}{(1-x) + \tilde{R}_s \left(\frac{\lambda}{(1-\lambda)} + \pi_s\right)^2 x} \\ & + (1 - \lambda) (1 - \pi_s) \frac{R_H}{R_H + \tilde{R}_s \left(\frac{\lambda}{(1-\lambda)} + \pi_s\right)} \end{aligned} \right) - x = 0$$

This is a monotonically decreasing function of x . At $x = 0$, F_b is greater than 0 and at $x = 1$, F_1 is less than zero. Therefore, by Intermediate Function Theorem, there exist a unique x^* such that at $F_1(x^*) = 0$ The x^* can be derived as a root to a cubic equation: $a_1 x^3 + a_2 x^2 + a_3 x + a_4 = 0$, where

$$\begin{aligned} a_1 &= -d_1 d_2 \\ a_2 &= d_1 d_2 d_3 - ((1 - \lambda) q_1 \pi_1 + 1) d_2 - ((1 - \lambda) q_2 \pi_2 + 1) d_1 \\ a_3 &= ((d_1 + d_2) d_3 - 1) + ((1 - \lambda) q_1 \pi_1 (d_2 - 1) + (1 - \lambda) q_2 \pi_2 (d_1 - 1)) \\ a_4 &= d_3 + (1 - \lambda) q_1 \pi_1 + (1 - \lambda) q_2 \pi_2 \end{aligned}$$

$$\begin{aligned}
d_1 &= \left(\tilde{R}_1 \left(\frac{\lambda}{(1-\lambda)} + \pi_1 \right)^2 - 1 \right) \\
d_2 &= \left(\tilde{R}_2 \left(\frac{\lambda}{(1-\lambda)} + \pi_2 \right)^2 - 1 \right) \\
d_3 &= \lambda \sum_{s=1,2} q_s \frac{1}{\left(\frac{1}{(1-\lambda)} + \pi_s \right)} + (1-\lambda) \sum_{s=1,2} q_s \frac{(1-\pi_s) R_H}{R_H + \tilde{R}_s \left(\frac{\lambda}{(1-\lambda)} + \pi_s \right)}
\end{aligned}$$

Denote the solution as x_b^* , then the prices are given by

$$p_b^*(s) = \frac{(1-\lambda)}{(\lambda + (1-\lambda)\pi_s)} \frac{(1-x_b^*)}{x_b^*}$$

If $p_b^*(s_2) \geq r$ then this is the equilibrium of type (1).

If $p_b^*(s_2) < r$ and $p_b^*(s_1) \geq r$ then consider type (2) equilibrium: type (1) equilibrium:

$$\begin{aligned}
\max_x & \left\{ \begin{aligned} & \lambda(1-q) \log(1-x+p_s x) \\ & + (1-\lambda)(1-q) \left(\begin{aligned} & \pi_1 \log(xp + (1-x)\tilde{R}_1/p_1) \\ & + (1-\pi_s) \log(xR_H + (1-x)\tilde{R}_1/p_1) \end{aligned} \right) + \\ & + \lambda q \log(1-x+rx) \\ & + (1-\lambda)q \left(\begin{aligned} & \pi_2 \log(xr + (1-x)) \\ & + (1-\pi_2) \log(xR_H + (1-x)) \end{aligned} \right) \end{aligned} \right\} \\
s.t. & \quad (i) \quad 0 \leq x \leq 1 \\
& \quad (ii) \quad p_1 \geq r
\end{aligned}$$

Therefore, the type 1 equilibrium investment allocation and market prices are

determined by the following equations:

$$\begin{aligned}
& (1 - q) \left(\lambda \frac{p_1 - 1}{1 - x + p_1 x} + (1 - \lambda) \left(\pi \frac{p_1 - \tilde{R}_1 / p_1}{x p_1 + (1 - x) \tilde{R}_1 / p_1} + (1 - \pi) \frac{R_H - \tilde{R}_1 / p_1}{x R_H + (1 - x) \tilde{R}_1 / p_1} \right) \right) = 0 \\
& + q_2 \left((\lambda + (1 - \lambda) \pi_2) \frac{r - 1}{x r + (1 - x)} + (1 - \pi_2) \frac{R_H - 1}{x R_H + (1 - x)} \right) \\
& (\lambda + (1 - \lambda) \pi_1) p_1 x = (1 - \lambda) (1 - x)
\end{aligned}$$

Substituting p_1 , we can get

$$G_b(x) \equiv \begin{pmatrix} q_1 \left(\lambda \frac{1}{\left(\frac{1}{(1-\lambda)} + \pi_1\right)} + (1 - \lambda) \pi \frac{(1-x)}{(1-x) + \tilde{R}_1 \left(\frac{\lambda}{(1-\lambda)} + \pi_s\right)^2 x} \right. \\ \left. + (1 - \lambda) (1 - \pi_1) \frac{R_H}{R_H + \tilde{R}_1 \left(\frac{\lambda}{(1-\lambda)} + \pi_1\right)} - x \right) \\ \left. + q_2 \left((\lambda + (1 - \lambda) \pi_2) \frac{r - 1}{x r + (1 - x)} + (1 - \pi_2) \frac{R_H - 1}{x R_H + (1 - x)} \right) \right) = 0
\end{pmatrix}$$

G_b is also a decreasing function in x , and it is positive at $x = 0$ and negative at $x = 1$. Therefore, the solution exists and it unique. Let x_b^{**} denote the solution, then the market price in state s_1 is given by

$$p_b^{**}(s_1) = \frac{(1 - \lambda)}{(\lambda + (1 - \lambda) \pi_1)} \frac{(1 - x_b^{**})}{x_b^{**}}$$

If $p_b^*(s_1) \geq r$ then this is the equilibrium of type (2). Note, $x_b^{**} < x_b^*$ since $F_1(x_b^{**}) > F_2(x_b^{**}) = 0$ and F_1 is decreasing in x .

If $p_b^*(s_1) < r$ then the equilibrium is of type (3). The type (3) no trade equilibrium is the same as no trade equilibrium considered in Proposition 1, and the equilibrium investment allocation is given by

$$x_a^{**} = \frac{(\lambda + (1 - \lambda) \bar{\pi}) (r - 1) + (1 - \lambda) (1 - \bar{\pi}) (R_H - 1)}{(1 - r) (R_H - 1)}$$

Furthermore, the investment allocation in the market equilibrium without adverse selection is larger than the investment allocation when adverse selection is present: $x_b^* < x_a^*$.

Let (x_a^*, p_a^*) be the equilibrium without adverse selection. Now consider the solution to maximization problem in Proposition 1 but with prices $p_b(s) = \frac{1}{\left(\frac{\lambda}{(1-\lambda)} + \pi_s\right)} \frac{1-x}{x}$ instead of $p_a = \frac{(1-\lambda)}{\lambda} \frac{(1-x)}{x}$. Denote the solution as $(x_a^o, p_b^o(s))$

$$\begin{aligned}
x_a^o &= \lambda \frac{1}{\left(\frac{\lambda}{(1-\lambda)} + \pi\right) + 1} + (1-\lambda) \sum_{s=1,2} q_s \left(\frac{\pi_s \frac{r}{r + \bar{R}_s \left(\frac{\lambda}{(1-\lambda)} + \pi_s\right)}}{+ (1-\pi_s) \frac{R_H}{R_H + \bar{R}_s \left(\frac{\lambda}{(1-\lambda)} + \pi_s\right)}} \right) \\
&< x_a^* \\
0 &= \sum_{s=1,2} q_s \left(\frac{\lambda \frac{p_b^o(s)-1}{1-x_a^o + p_b^o(s)x_a^o}}{+ (1-\lambda) \left(\frac{\pi_s \frac{r - \bar{R}_s/p_b^o(s)}{x_a^o r + (1-x_a^o)\bar{R}_s/p_b^o(s)}}{+ (1-\pi_s) \frac{R_H - \bar{R}_s/p_b^o(s)}{x_a^o R_H + (1-x_a^o)\bar{R}_s/p_b^o(s)}} \right)} \right) < \\
&< \left(\frac{\lambda \frac{p_b^o(s)-1}{1-x_a^o + p_b^o(s)x_a^o}}{+ (1-\lambda) \sum_{s=1,2} q_s \left(\frac{\pi_s \frac{p_b^o(s) - \tilde{R}_s/p_b^o(s)}{x_a^o p_b^o(s) + (1-x_a^o)\tilde{R}_s/p_b^o(s)}}{+ (1-\pi_s) \frac{R_H - \tilde{R}_s/p_b^o(s)}{x_a^o R_H + (1-x_a^o)\tilde{R}_s/p_b^o(s)}} \right)} \right) = F_1(x_a^o)
\end{aligned}$$

Therefore, $x_b^* < x_a^o$ such that $F_1(x_b^*) = 0$. Hence, $x_b^* < x_a^o < x_a^*$.

Also, adverse selection lead to a lower expected price: $p_a > p_b((1-q)s_1 + qs_2) \geq (1-q)p_b(s_1) + qp_b(s_2)$

$$p_a > (1-q)p_b(s_1) + qp_b(s_2)$$

In the presence of adverse selection, the highest utility is attained when there a market trading in equilibrium in both states. The equilibrium consumption of

early and late consumers are given by

$$\begin{aligned} c_1^b(s) &= 1 - x_b + p_b(s) x_b \\ c_{2H}^b(s) &= x_b R_H + (1 - x_b) \tilde{R}_s / p_b(s) \\ c_{2L}^b(s) &= x_b p_b + (1 - x_b) \tilde{R}_s / p_b(s) \end{aligned}$$

The expected consumption at both dates in the equilibrium with adverse selection are lower than the expected consumption at both dates without adverse selection. Therefore, expected utility is lower. In case of adverse selection the low quality projects do not get liquidated by informed investors. This results in the losses of total welfare. ■

B.2.2 Proof of Corollary 1

Proof. First consider an equilibrium with trade in both states.

The equilibrium investment allocation is determined from the following equation: $F_b(x) = 0$ ($F_b(x)$ is defined in the proof of Proposition 2, it is derived by substituting market clearing conditions into the FOC condition.) Denote by $F_{b1}(s)$ the following expression,

$$\begin{aligned} F_{b1}(s, x) \equiv & \lambda \frac{1}{\left(\frac{1}{(1-\lambda)} + \pi_s\right)} + (1 - \lambda) \pi_s \frac{(1-x)}{(1-x) + \tilde{R}_s \left(\frac{\lambda}{(1-\lambda)} + \pi_s\right)^2} x \\ & + (1 - \lambda) (1 - \pi_s) \frac{R_h}{R_h + \tilde{R}_s \left(\frac{\lambda}{(1-\lambda)} + \pi_s\right)} - x. \end{aligned}$$

$F_{b1}(s) = 0$ provides the solution for the problem with one state. $F_{b1}(s)$ is decreasing in π_s . Therefore, $F_b(x)$ is decreasing in q . Also, $F_b(x)$ is decreasing in x . Hence, x is decreasing in q . The prices are determined by $p_s = \frac{(1-\lambda)}{(\lambda + (1-\lambda)\pi_s)} \frac{(1-x)}{x}$.

Therefore, p_s are increasing in q . The one-state expected utility is decreasing in π_s . Therefore, as q becomes larger the expected utility decreases.

Now consider an equilibrium with a no trade state 2. Denote by $G_{b2}(x) = \left((\lambda + (1 - \lambda) \pi_2) \frac{r-1}{xr+(1-x)} + (1 - \pi_2) \frac{R_H-1}{xR_H+(1-x)} \right)$. If we compute x^* such that $G_{b2}(x^*) = 0$ and x^{**} such that $F_{b1}(s = 1, x^*) = 0$ then $x^{**} > x^*$. The equilibrium x in a two-state problem is determined by $G_b(x) = (1-q)F_{b1}(s = 1, x) + qG_{b2}(x) = 0$. Since $G_b(x)$ is decreasing in x then the optimal x is decreasing in q . Therefore, p_1 are increasing in q since it negatively depends on x . The one-state expected utility is lower in a no-trade state vs the one with trade. Therefore, as q becomes larger the expected utility decreases. The no-trade outcome arises since the price in the crisis state falls below liquidation value. The increase in q may increase the price in the crisis state sufficiently to restore the trading.

Consider some q such that $p_2 = r - \varepsilon$ with $\varepsilon > 0$. Then there is no trading in state 2.

$$\begin{aligned} F_{b1}(s = 1, x) &= \lambda \frac{1}{\left(\frac{1}{(1-\lambda)} + \pi_1\right)} + (1 - \lambda) \pi_1 \frac{1}{1 + \tilde{R}_1 \left(\frac{\lambda}{(1-\lambda)} + \pi_1\right)^2 (r-\varepsilon) / \left(\frac{\lambda}{(1-\lambda)} + \pi_2\right)} \\ &\quad + (1 - \lambda) (1 - \pi_1) \frac{R_h}{R_h + \tilde{R}_1 \left(\frac{\lambda}{(1-\lambda)} + \pi_2\right)} - \frac{1}{\left(1 + \left(\frac{\lambda}{(1-\lambda)} + \pi_2\right)(r+\varepsilon)\right)} \\ F_{b1}(s = 2, x) &= \lambda \frac{1}{\left(\frac{1}{(1-\lambda)} + \pi_2\right)} + (1 - \lambda) \pi_1 \frac{1}{1 + \tilde{R}_2 \left(\frac{\lambda}{(1-\lambda)} + \pi_2\right)(r-\varepsilon)} \\ &\quad + (1 - \lambda) (1 - \pi_1) \frac{R_h}{R_h + \tilde{R}_2 \left(\frac{\lambda}{(1-\lambda)} + \pi_2\right)} - \frac{1}{\left(1 + \left(\frac{\lambda}{(1-\lambda)} + \pi_2\right)(r+\varepsilon)\right)} \end{aligned}$$

Therefore, $F_{b1}(s = 1, x) > F_{b1}(s = 2, x)$. If q increases sufficiently so that x goes down by more than $\frac{\left(\frac{\lambda}{(1-\lambda)} + \pi_2\right)\varepsilon}{\left(1 + \left(\frac{\lambda}{(1-\lambda)} + \pi_2\right)(r+\varepsilon)\right)\left(1 + \left(\frac{\lambda}{(1-\lambda)} + \pi_2\right)r\right)}$ then the trading in a crisis state restores. ■

B.2.3 Proof of Corollary 2

Proof. Suppose now the economy is parametrized by state 1: (λ_1, π_1) and state 2: (λ_2, π_2) such that $\lambda_1 < \lambda_2$ and $\pi_1 < \pi_2$.

First consider an equilibrium with trade in both states. The equilibrium investment allocation is determined from the following equation: $F_b(x) = 0$

Denote by $F_{b1}(s)$ the following expression,

$$F_{b1}(s, x) \equiv \lambda \frac{1}{\left(\frac{1}{(1-\lambda)} + \pi_s\right)} + (1-\lambda) \pi_s \frac{(1-x)}{(1-x) + \tilde{R}_s \left(\frac{\lambda}{(1-\lambda)} + \pi_s\right)^2} x \\ + (1-\lambda) (1-\pi_s) \frac{R_h}{R_h + \tilde{R}_s \left(\frac{\lambda}{(1-\lambda)} + \pi_s\right)} - x$$

$F_{b1}(s) = 0$ provides the solution for the problem with one state. $F_{b1}(s)$ is decreasing in λ . Also, $F_b(x)$ is decreasing in x . Hence, x is decreasing in λ .

The effect of increase in λ_2 on the price in state 2 is determined by

$$\frac{\partial p_2}{\partial \lambda_2} = - \frac{\frac{1}{(1-\lambda_2)^2}}{\left(\frac{\lambda_2}{1-\lambda_2} + \pi_2\right)^2} \frac{(1-x)}{x} - \frac{1}{\left(\frac{\lambda_2}{1-\lambda_2} + \pi_2\right)} \frac{1}{x^2} \frac{\partial x}{\partial \lambda_2}$$

Therefore, increase in λ_2 can lead to the decrease in p_2 , potentially resulting in $p_2 < r$. ■

B.3 Central Planner Allocation

B.3.1 Liquidity Shock is Public Information

Proof.

$$\begin{aligned}
 EU(c_1, c_2) &= \lambda \log c_1 + (1 - \lambda) \sum_s q_s (\pi_s \log(c_{2Ls}) + (1 - \pi_s) \log(c_{2Hs})) \\
 s.t. \quad &: c_1 = \frac{1 - x}{\lambda} \\
 &: c_{2L} = x \left(r + \bar{R} \frac{\lambda}{1 - \lambda} \right) \\
 &: c_{2H} = x \left(R_H + \bar{R} \frac{\lambda}{1 - \lambda} \right) \\
 IC \quad &: x \bar{R} \frac{\lambda}{1 - \lambda} \geq 1 - x
 \end{aligned}$$

$\Rightarrow x^o = (1 - \lambda)$. The incentive compatibility constraint is satisfied since $(1 - \lambda)(\pi r + (1 - \pi) R_H) + \lambda \bar{R} > 1$. Therefore, consumption allocations are given by

$$\begin{aligned}
 c_1 &= 1 \\
 c_{2L}(s) &= (1 - \lambda) \left(r + \bar{R}_s \frac{\lambda}{1 - \lambda} \right) \\
 c_{2H}(s) &= (1 - \lambda) \left(R_H + \bar{R}_s \frac{\lambda}{1 - \lambda} \right)
 \end{aligned}$$

■

B.3.2 Liquidity Shock is Private Information

Proof. additional constraint:

$$c_{2L2} \geq c_1$$

$$(1 - \lambda) r + \lambda \bar{R} \geq 1$$

if $(1 - \lambda) r + \lambda \bar{R} \geq 1$ then no late consumer has incentive to pretend to be an early one $\implies x^o = (1 - \lambda)$ if $(1 - \lambda) r + \lambda \bar{R} < 1$ then x is determined by $c_1 = c_{2L2}$, hence,

$$\begin{aligned} \frac{1 - x}{\lambda} &= x \left(r + \bar{R} \frac{\lambda}{1 - \lambda} \right) \\ x^{oo} &= \frac{1 - \lambda}{((1 - \lambda) + \lambda ((1 - \lambda) r + \lambda \bar{R}_2))} = x \\ x^{oo} &= \frac{1 - \lambda}{(1 - \lambda [1 - (1 - \lambda) r + \lambda \bar{R}_2])} > 1 - \lambda \end{aligned}$$

Note, $x^{oo} < x^o$

$$\begin{aligned} \frac{1}{(1 + \frac{\lambda}{1 - \lambda} ((1 - \lambda) r + \lambda \bar{R}_2))} &< (1 - \lambda) \\ 1 &< (1 - \lambda) + \lambda ((1 - \lambda) r + \lambda \bar{R}_2) \end{aligned}$$

Therefore,

$$c_1^{oo} = c_{2L}^{oo} = \frac{\frac{\lambda}{1 - \lambda} ((1 - \lambda) r + \lambda \bar{R}_2)}{1 + \frac{\lambda}{1 - \lambda} ((1 - \lambda) r + \lambda \bar{R}_2)}$$

■

B.3.3 Central Planner vs Market Allocations

Proof.

$$\begin{aligned} EU(x^o) &= (1 - \lambda) \sum_s q_s \left(\begin{aligned} &\pi_s \log((1 - \lambda) (r + \bar{R}_s \frac{\lambda}{1 - \lambda})) \\ &+ (1 - \pi_s) \log((1 - \lambda) (R_H + \bar{R}_s \frac{\lambda}{1 - \lambda})) \end{aligned} \right) = \\ &= (1 - \lambda) \log(1 - \lambda) + (1 - \lambda) \sum_s q_s \left(\begin{aligned} &\pi_s \log(r + \bar{R}_s \frac{\lambda}{1 - \lambda}) \\ &+ (1 - \pi_s) \log(R_H + \bar{R}_s \frac{\lambda}{1 - \lambda}) \end{aligned} \right), \end{aligned}$$

$$\begin{aligned}
EU(x_a) &= \lambda \log \frac{(1-x)}{\lambda} + (1-\lambda) \sum_s q_s \left(\begin{array}{c} \pi_s \log \left(x \left(R_i + \bar{R}_s \frac{\lambda}{1-\lambda} \right) \right) \\ + (1-\pi_s) \log \left(x \left(R_i + \bar{R}_s \frac{\lambda}{1-\lambda} \right) \right) \end{array} \right) = \\
&= \left(\begin{array}{c} \lambda \log \frac{(1-x)}{\lambda} + (1-\lambda) \log \frac{x}{(1-\lambda)} \\ + (1-\lambda) \log (1-\lambda) + (1-\lambda) \sum_s q_s \left(\begin{array}{c} \pi_s \log \left(R_i + \bar{R}_s \frac{\lambda}{1-\lambda} \right) \\ + (1-\pi_s) \log \left(R_i + \bar{R}_s \frac{\lambda}{1-\lambda} \right) \end{array} \right) \end{array} \right),
\end{aligned}$$

therefore, $EU(x_a) - EU(x^o) = \lambda \log \frac{(1-x)}{\lambda} + (1-\lambda) \log \frac{x}{(1-\lambda)}$

Claim: $x_a \geq x^o = 1 - \lambda$

$$\begin{aligned}
x_a &= \lambda(1-\lambda) + (1-\lambda) \sum_{s=1,2} q_s \left(\begin{array}{c} \pi_s \frac{r}{(r + \bar{R}_s \frac{\lambda}{1-\lambda})} \\ + (1-\pi_s) \frac{R_H}{(R_H + \bar{R}_s \frac{\lambda}{1-\lambda})} \end{array} \right) \\
&\leq 1 - \lambda \\
\lambda + \sum_{s=1,2} q_s \left(\pi_s \frac{r}{(r + \bar{R}_s \frac{\lambda}{1-\lambda})} + (1-\pi_s) \frac{R_H}{(R_H + \bar{R}_s \frac{\lambda}{1-\lambda})} \right) &\geq 1
\end{aligned}$$

since $\sum_{s=1,2} q_s \left[\left(r - \bar{R}_s \right) - (1-\lambda) (1-\pi_s) \frac{(R_H - R_L)(R_H - r)}{(1-\lambda)R_H + \lambda \bar{R}_s} \right] \geq 0$. Hence,

$EU(x_a) - EU(x^o) = \lambda \log \frac{(1-x)}{\lambda} + (1-\lambda) \log \frac{x}{(1-\lambda)} \geq 0$ since $x_a \geq (1-\lambda)$. \blacksquare

APPENDIX C

APPENDIX TO CHAPTER 3

C.1 No Uncertainty about Asset Payoffs

The investor's decision problem if there is no uncertainty about distribution of asset returns:

$$\max\{(\bar{r}_a - p_a)x_a + (\bar{r}_b - p_b)x_b + e - \frac{\gamma}{2}(\sigma_a^2 x_a^2 + \sigma_b^2 x_b^2)\} \quad (\text{C.1})$$

The optimal demand for risky assets $k = a, b$:

$$x_k^o = \frac{\bar{r}_k - p_k}{\gamma \sigma_k^2} \quad (\text{C.2})$$

The equilibrium prices resulting from market clearing conditions:

$$p_k^o = \bar{r}_k - \lambda_k \bar{x}_k \gamma \sigma_k^2 \quad (\text{C.3})$$

The equilibrium demand for risky asset is equal to its market share

$$x_k^o = \lambda_k \bar{x}_k \quad (\text{C.4})$$

C.2 Uncertainty about Asset Payoffs

The distribution of asset k returns, there is uncertainty (ambiguity) about mean returns

$$r_k \sim N(\bar{r}_{ks}, \sigma_k^2) \quad (\text{C.5})$$

$$\bar{r}_{ks} \sim N(\bar{r}_k, \delta_k^2)$$

The investor's decision problem in the presence of uncertainty about distribution of both asset returns

$$\max \alpha E \left[E[w|s] - \frac{1}{2} \text{var}[w|s] \right] - \frac{1}{2} \alpha^2 \text{Var} \left[E[w|s] - \frac{1}{2} \text{var}[w|s] \right] \quad (\text{C.6})$$

$$(i) \text{Cov}(\bar{r}_a, \bar{r}_b) = \delta_{ab} \neq 0$$

$$\max \left\{ \begin{array}{l} \alpha \gamma (\bar{r}_a - p_a) x_a + \alpha \gamma (\bar{r}_b - p_b) x_b + \alpha \gamma e \\ -\frac{\alpha \gamma^2}{2} (\sigma_a^2 x_a^2 + \sigma_b^2 x_b^2) - \frac{\alpha^2 \gamma^2}{2} (\delta_a^2 x_a^2 + \delta_b^2 x_b^2 + 2\delta_{ab} x_a x_b) \end{array} \right\} \quad (\text{C.7})$$

Optimal demand for risky assets:

$$\begin{aligned} x_a &= \frac{(\bar{r}_a - p_a) (\sigma_b^2 + \alpha \delta_b^2) - (\bar{r}_b - p_b) \alpha \delta_{ab}^2}{\gamma [(\sigma_a^2 \sigma_b^2 + \alpha \sigma_a^2 \delta_b^2 + \alpha \sigma_b^2 \delta_a^2)]} \\ x_b &= \frac{(\bar{r}_b - p_b) (\sigma_a^2 + \alpha \delta_a^2) - (\bar{r}_a - p_a) \alpha \delta_{ab}}{\gamma [(\sigma_a^2 \sigma_b^2 + \alpha \sigma_a^2 \delta_b^2 + \alpha \sigma_b^2 \delta_a^2)]} \end{aligned} \quad (\text{C.8})$$

Equilibrium prices:

$$\begin{aligned} \tilde{p}_a &= \bar{r}_a - \lambda \bar{x}_a \gamma (\sigma_a^2 + \alpha \delta_a^2) - (1 - \lambda) \bar{x}_b \gamma \alpha \delta_{ab} \\ \tilde{p}_b &= \bar{r}_b - (1 - \lambda) \bar{x}_b \gamma (\sigma_b^2 + \alpha \delta_b^2) - \lambda \bar{x}_a \gamma \alpha \delta_{ab} \end{aligned} \quad (\text{C.9})$$

(ii) returns distributions are independent across states than $\text{Cov}(\bar{r}_a, \bar{r}_b) = \delta_{ab} = 0$

The optimal demand for assets:

$$\begin{aligned} x_a &= \frac{(\bar{r}_a - p_a) (\sigma_b^2 + \alpha \delta_b^2)}{\gamma [(\sigma_a^2 \sigma_b^2 + \alpha \sigma_a^2 \delta_b^2 + \alpha \sigma_b^2 \delta_a^2)]} \\ x_b &= \frac{(\bar{r}_b - p_b) (\sigma_a^2 + \alpha \delta_a^2)}{\gamma [(\sigma_a^2 \sigma_b^2 + \alpha \sigma_a^2 \delta_b^2 + \alpha \sigma_b^2 \delta_a^2)]} \end{aligned} \quad (\text{C.10})$$

Equilibrium prices:

$$\begin{aligned}\tilde{p}_a &= \bar{r}_a - \lambda \bar{x}_a \gamma (\sigma_a^2 + \alpha \delta_a^2) \\ \tilde{p}_b &= \bar{r}_b - (1 - \lambda) \bar{x}_b \gamma (\sigma_b^2 + \alpha \delta_b^2)\end{aligned}\tag{C.11}$$

C.3 Proof of Proposition 1

Proof. Investors are optimistic ($\bar{r}_{ah} > \bar{r}_{af}$) and overconfident ($\delta_{ah} < \delta_{af}$) about the home asset relative to the foreign. Then the decision problem of investor i from country A :

$$\begin{aligned}& \max \alpha E \left[E[w|s_n] - \frac{1}{2} \text{var}[w|s_n] \right] - \frac{1}{2} \alpha^2 \text{Var} \left[E[w|s_n] - \frac{1}{2} \text{var}[w|s_n] \right] \\ & \max \left\{ (\bar{r}_{ah} - p_a)x_a + (\bar{r}_{bf} - p_b)x_b - \frac{\gamma}{2}(\sigma_a x_a^2 + \sigma_b x_b^2) - \frac{\alpha \gamma}{2}(\delta_{ah} \gamma^2 x_a^2 + \delta_{bf} \gamma^2 x_b^2) \right\} \\ & \bar{r}_{ah} - p_a - \gamma \sigma_a x_a - \alpha \gamma \delta_{ah} x_a = 0 \\ & \bar{r}_{bf} - p_b - \gamma \sigma_b x_b - \alpha \gamma \delta_{bf} x_b = 0 \\ & x_{ai}^* = \frac{(\bar{r}_{ah} - p_a)}{\gamma(\sigma_a + \alpha \delta_{ah})} \quad x_{bi}^* = \frac{(\bar{r}_{bf} - p_b)}{\gamma(\sigma_b + \alpha \delta_{bf})} \\ & x_{aj}^* = \frac{(\bar{r}_{af} - p_a)}{\gamma(\sigma_a + \alpha \delta_{af})} \quad x_{bj}^* = \frac{(\bar{r}_{bh} - p_b)}{\gamma(\sigma_b + \alpha \delta_{bh})}\end{aligned}$$

Market clearing conditions:

$$\begin{aligned}\lambda x_{ai}^* + (1 - \lambda) x_{aj}^* &= \lambda \bar{x}_a \\ \lambda x_{bi}^* + (1 - \lambda) x_{bj}^* &= (1 - \lambda) \bar{x}_b\end{aligned}$$

Equilibrium asset prices:

$$\begin{aligned}p_a^* &= \frac{\lambda \bar{r}_{ah}(\sigma_a + \alpha \delta_{af}) + (1 - \lambda) \bar{r}_{af}(\sigma_a + \alpha \delta_{ah})}{\sigma_a + \alpha(\lambda \delta_{af} + (1 - \lambda) \delta_{ah})} - \lambda \bar{x}_a \gamma \frac{(\sigma_a + \alpha \delta_{ah})(\sigma_a + \alpha \delta_{af})}{\sigma_a + \alpha(\lambda \delta_{af} + (1 - \lambda) \delta_{ah})} ; \\ p_b^* &= \frac{(1 - \lambda) \bar{r}_{bh}(\sigma_b + \alpha \delta_{bf}) + \lambda \bar{r}_{bf}(\sigma_b + \alpha \delta_{bh})}{\sigma_b + \alpha(\lambda \delta_{bh} + (1 - \lambda) \delta_{bf})} - (1 - \lambda) \bar{x}_b \gamma \frac{(\sigma_b + \alpha \delta_{bh})(\sigma_b + \alpha \delta_{bf})}{\sigma_b + \alpha(\lambda \delta_{bh} + (1 - \lambda) \delta_{bf})} .\end{aligned}$$

Equilibrium portfolio holdings for investor i from country A :

$$\begin{aligned}
x_{ai}^* &= \frac{\lambda \bar{x}_a \frac{(\sigma_a + \alpha \delta_{af})}{\sigma_a + \alpha (\lambda \delta_{af} + (1-\lambda) \delta_{ah})}}{\frac{(1-\lambda)(\bar{r}_{ah} - \bar{r}_{af})}{\gamma \sigma_a + \alpha (\lambda \delta_{af} + (1-\lambda) \delta_{ah})}} > \lambda \bar{x}_a \\
x_{bi}^* &= \frac{(1-\lambda) \bar{x}_b \frac{(\sigma_b + \alpha \delta_{bh})}{\sigma_b + \alpha (\lambda \delta_{bh} + (1-\lambda) \delta_{bf})}}{\frac{\lambda (\bar{r}_{bh} - \bar{r}_{bf})}{\gamma \sigma_b + \alpha (\lambda \delta_{bh} + (1-\lambda) \delta_{bf})}} < (1-\lambda) \bar{x}_b
\end{aligned}$$

Asset Prices

expected price by home investors :

$$\tilde{p}_{ah} = \bar{r}_{ah} - \bar{x}_a \gamma (\sigma_a + \alpha \delta_{ah})$$

expected price by foreign investors :

$$\tilde{p}_{af} = \bar{r}_{af} - \bar{x}_a \gamma (\sigma_a + \alpha \delta_{af})$$

equilibrium price :

$$\begin{aligned}
p_a^* &= \left(\frac{\bar{r}_{ah} \lambda (\sigma_a + \alpha \delta_{af}) + \bar{r}_{af} (1-\lambda) (\sigma_a + \alpha \delta_{ah})}{[\sigma_a + \alpha (\lambda \delta_{af} + (1-\lambda) \delta_{ah})]} \right. \\
&\quad \left. - \bar{x}_a \gamma \frac{(\sigma_a + \alpha \delta_{ah}) (\sigma_a + \alpha \delta_{af})}{[\sigma_a + \alpha (\lambda \delta_{af} + (1-\lambda) \delta_{ah})]} \right) \\
\tilde{p}_{ah} &> p_a^* > \tilde{p}_{af}
\end{aligned}$$

■

C.4 Proof of Proposition 2

Proof. Equilibrium portfolio holdings for investor i from country A :

$$\begin{aligned}
x_{ai}^* &= \lambda \bar{x}_a \frac{\sigma_a + \alpha (\Delta \delta_a + \delta_{ah})}{\sigma_a + \alpha (\lambda \Delta \delta_a + \delta_{ah})} + \frac{(1-\lambda) \Delta \bar{r}_a}{\gamma [\sigma_a + \alpha (\lambda \Delta \delta_a + \delta_{ah})]} \\
x_{bi}^* &= (1-\lambda) \bar{x}_b \frac{(\sigma_b + \alpha (\Delta \delta_b + \delta_{bh}))}{[\sigma_b + \alpha (\lambda \Delta \delta_b + \delta_{bh})]} - \frac{\lambda \Delta \bar{r}_b}{\gamma [\sigma_b + \alpha (\lambda \Delta \delta_b + \delta_{bh})]}
\end{aligned}$$

degree of ambiguity aversion α : If the degree of ambiguity aversion increases then the prices of both assets go down, the holding of the home asset increases and the

foreign asset holding decreases. Hence, the equity home bias becomes larger

$$\begin{aligned}\frac{\partial x_{ai}^*}{\partial \alpha} &= \frac{-\lambda \bar{x}_a \gamma^2 (1 - \lambda) \Delta \delta_a + \lambda \Delta \delta_a + \delta_{ah}}{\gamma^2 (\sigma_a + \alpha (\lambda \Delta \delta_a + \delta_{ah}))^2} \\ \frac{\partial x_{bi}^*}{\partial \alpha} &= \frac{-(1 - \lambda) \bar{x}_b \gamma^2 \lambda \Delta \delta_b - \lambda \Delta \delta_b - \delta_{bh}}{\gamma^2 [\sigma_b + \alpha (\lambda \Delta \delta_b + \delta_{bh})]^2} < 0\end{aligned}$$

If the degree of ambiguity aversion increases then the home asset holdings may increase or decrease and the foreign asset holding decreases. Overall, the equity home bias becomes larger.

- difference in perceived dispersion of mean asset returns $\Delta \delta_k = \delta_{kf} - \delta_{kh}$:

$$\begin{aligned}\frac{\partial x_{ai}^*}{\partial \Delta \delta_a} &= \frac{\alpha \lambda (1 - \lambda) \Delta \bar{r}_a}{\gamma^2 [\sigma_a + \alpha (\lambda \Delta \delta_a + \delta_{ah})]^2} > 0 \\ \frac{\partial x_{bi}^*}{\partial \Delta \delta_b} &= \frac{-\alpha (1 - \lambda) \lambda \Delta \bar{r}_b}{\gamma^2 ([\sigma_b + \alpha (\lambda \delta_{bh} + (1 - \lambda) \delta_{bf})]^2)} < 0\end{aligned}$$

If for a given asset difference in perceived dispersion of mean asset returns $\Delta \delta_k$ increases then the holding of this asset decreases if it is a foreign asset, and increases if it is a home asset. Therefore, the equity home bias becomes larger.

- difference in perceived mean returns of mean asset returns $\Delta \bar{r}_k = \bar{r}_{kh} - \bar{r}_{kf}$
:

$$\begin{aligned}\frac{\partial x_{ai}^*}{\partial \Delta \bar{r}_a} &= \frac{(1 - \lambda)}{\gamma [\sigma_a + \alpha (\lambda \Delta \delta_a + \delta_{ah})]} > 0 \\ \frac{\partial x_{bi}^*}{\partial \Delta \bar{r}_b} &= \frac{-\lambda}{\gamma ([\sigma_b + \alpha (\lambda \delta_{bh} + (1 - \lambda) \delta_{bf})]} < 0\end{aligned}$$

If for a given asset difference in perceived mean returns $\Delta \bar{r}_k$ increases then the holding of this asset decreases if it is a foreign asset, and increases if it is a home asset. Therefore, the equity home bias becomes larger.

- population fraction λ

$$\begin{aligned}\frac{\partial x_{ai}^*}{\partial \lambda} &= \frac{\gamma \bar{x}_a [\sigma_a + \alpha \delta_{af}] - \Delta \bar{r}_a}{\gamma [\sigma_a + \alpha (\lambda \Delta \delta_a + \delta_{ah})]} < 0 \\ \frac{\partial x_{bi}^*}{\partial \lambda} &= \frac{-\gamma \bar{x}_b [\sigma_b + \alpha \delta_{bf}] - \Delta \bar{r}_b}{\gamma [\sigma_b + \alpha (\lambda \Delta \delta_b + \delta_{bh})]} < 0\end{aligned}$$

If size of the population in country A decreases relative to country B then the equilibrium price for asset a goes up and the equilibrium price for asset b goes down. In country A, the home asset holdings increase and the foreign asset holdings decrease; vice versa for country B. Therefore, the equity home bias becomes larger.

■

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